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Discussion

Author's closure to J.T. Kirby's discussion 'Note on a nonlinearity parameter of surface waves'

S. Beji

Department of Naval Architecture and Ocean Engineering, Istanbul Technical University, Maslak 80626, Istanbul, Turkey

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1. General

The main purpose of the previous work (Beji, 1995), as its title made it perfectly clear, was to introduce a unified nonlinearity parameter that would be equally valid for both deep and shallow water waves. The outcome of this attempt, which remains undisputed, has been

$$\epsilon = ga/c^2, \tag{1}$$

where g is the gravitational acceleration, a the wave amplitude, and c the wave celerity as dictated by linear theory.

Kirby (1998) focuses on a somewhat different problem of obtaining a unified non-dimensional description of the water wave phenomenon, which yields the usual short and long wave scalings of the governing equations. Despite the lengthy arguments and the confusing aspects of an altered notation (compared with the work under discussion), the newly proposed scaling differs from the previous scaling only with respect to the vertical scale, which is determined through the use of Eq. (1). To state concisely, c^2/g is used as the vertical length scale instead of $1/k$, which was adopted in Beji (1995) as the simplest possible scale for the purpose.

2. Principal aspects of previous and present scalings

The prominent feature of the previous scaling (Beji, 1995) was the adoption of, what might be termed, a *dynamic* scaling variable; namely, the wave celerity c as given by

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linear theory. In contrast to the standard approaches, such a choice introduces a parameter that varies uniformly with depth, thus making a unified description possible. Use of this variable may also be viewed as an iteration (i.e., a result of linear wave theory used for a scaling). Kirby, who uses exactly the same scaling approach (except for z), writes of a 'natural' appearance of ϵ (α in his notation) in the governing equations and appears not to appreciate this crucial point. Without the use of c as a scaling variable, which is completely absent in the standard scalings, it would be impossible to obtain any of the results presented in Kirby (1998). This point also serves for an answer to the question raised by Kirby as to why the parameter ε had not been utilized earlier.

The appearance of $\varepsilon/\varepsilon_{\mathrm{d}}$ in the original scaling is pronounced a 'defect' and 'complication'. Obviously, the final form of this ratio has been overlooked, as it turns out to be

$$\varepsilon/\varepsilon_d = (ga/c^2)/(ka) = g/(kc^2) = 1/\tanh kh, \tag{2}$$

which is $1/\tilde{\mu}$ in Kirby's notation, a variable freely used in his final set of equations.

The non-dimensionalized equations in their original form have an unusual feature that they do not reduce to the equations that correspond to the long-wave scaling. It is, however, equally obvious that without a priori knowledge of ε, the *dynamic* scaling of the vertical coordinate would not be possible. [1] In this sense, Kirby's change of z-scale may be viewed as an extension of the previous work to provide a unified scaling of the governing equations.

3. Related works

In the original work, it was pointed out that Eq. (1) may be related to Miche's breaking criterion. Kirby makes a rather far fetched connection to the parameter of Cokelet (1977).

Some time after the publication of the note, the author became aware of the existence of a work by Banner and Phillips (1974) on the breaking of waves in presence of a surface drift. Pursuing an elegant line of thought, Banner and Phillips arrive at the following expression

$$\zeta_{\max} = (c^2/2g)(1 - q/c)^2, \tag{3}$$

where $\zeta_{\max}$ is the maximum wave height possible without breaking and q the surface drift. For $q = 0$, Eq. (3) is virtually identical with Miche's criterion (except for the constant) and it is relation to Eq. (1) is obvious. In view of such a clear connection, Banner and Phillips must too be given due credit.

[1] Of course, once the result is known, it is always possible to present the arguments in a way that would suggest that no prior knowledge of Eq. (1) is necessary.

References

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