

SHIP THEORY - DYNAMICS

SEAKEEPING AND
MANEUVERING

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Introduction

The main problem to consider:

How does a ship behave in waves?



- Safety of crew
- Comfort of passengers
- Effects of dynamics loads
- Speed sustainability



Introduction

- Comfort of passengers: There is a limit for a person to stand against a certain acceleration at a certain period. If the threshold is exceeded the person becomes sick.

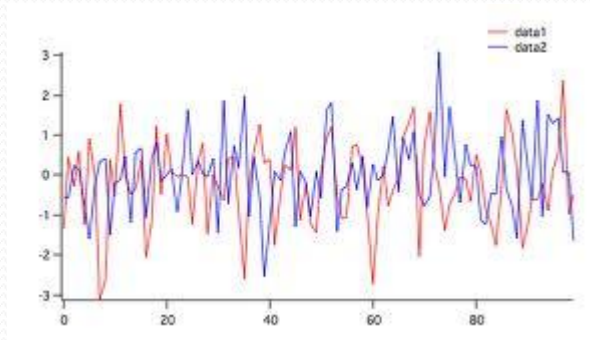
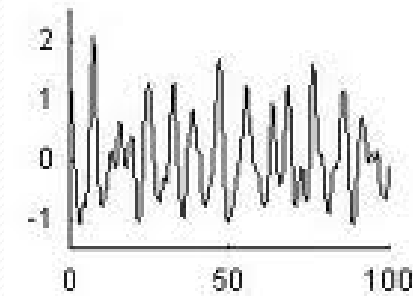


- Effects of dynamic loads: Large slamming forces may be quite damaging hence dangerous.

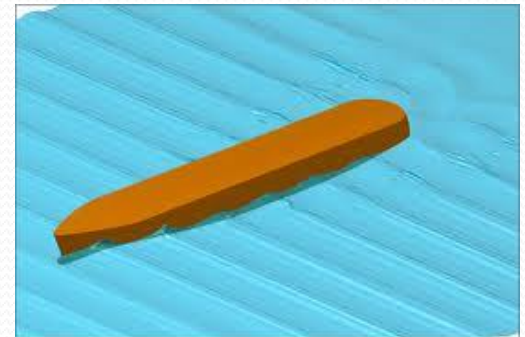
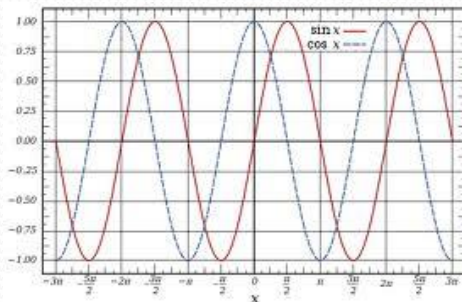


Regular Waves

- In order to study such effects we must first study real ocean waves, which are quite irregular or random in nature.

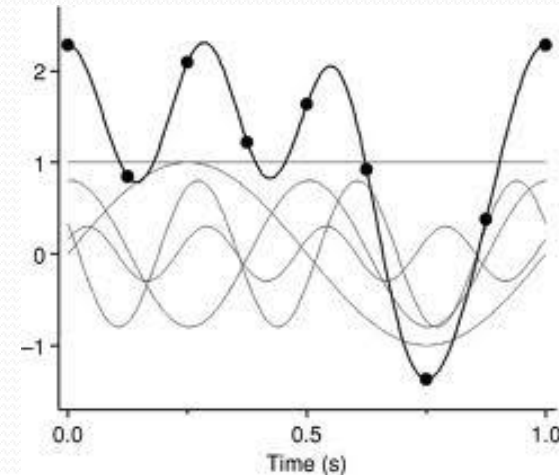
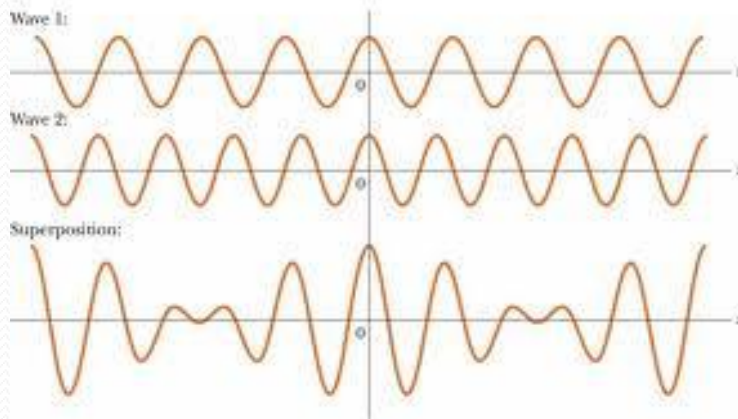


- However, before understanding and modeling real ocean waves we must begin with regular waves, which are quite deterministic and sinusoidal in form.



Regular Waves

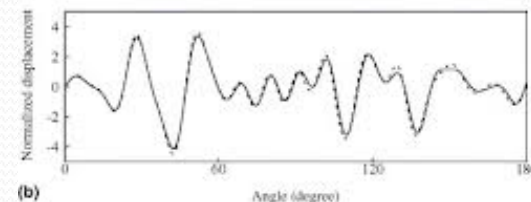
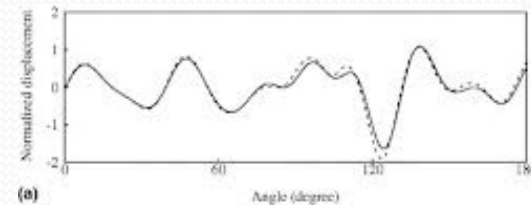
- Linear random waves may be constructed from the superposition of linear sinusoidal waves.



- Sum of two sinusoids (left), five sinusoids (right).

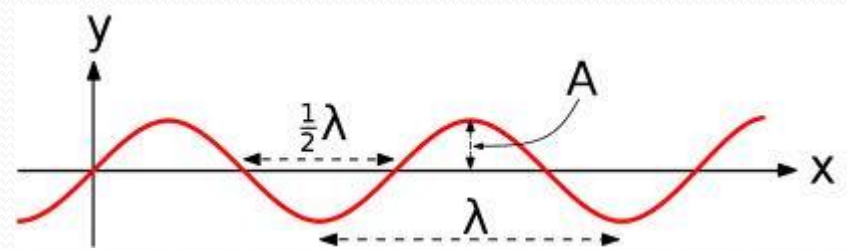
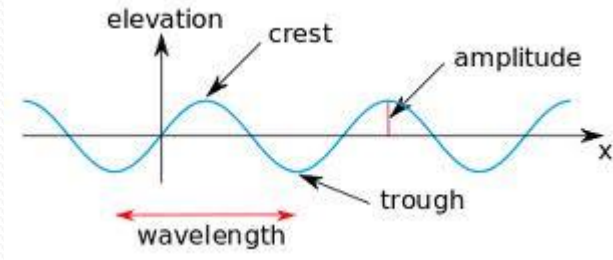


- Sum of tens of sinusoids.

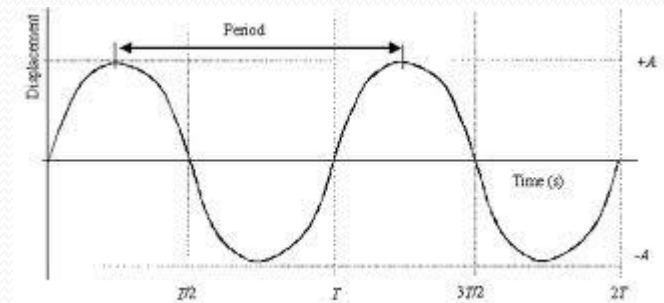
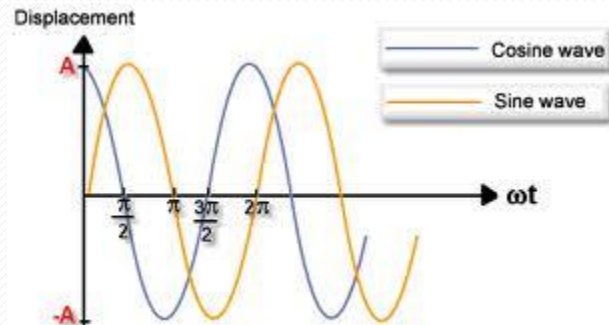


Regular Waves

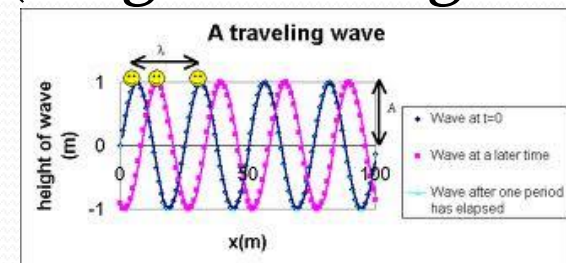
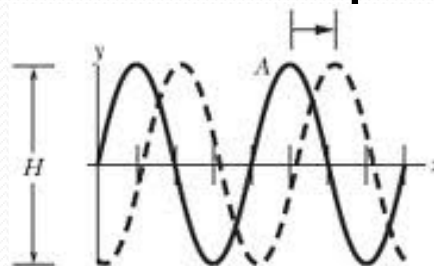
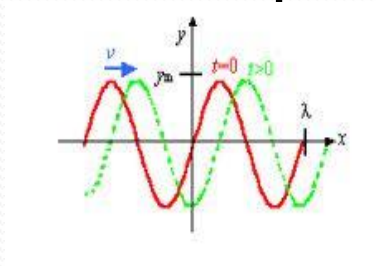
- 1-D sinusoidal wave in space is viewed as



- It can also be viewed in time



- It moves in space as time elapses (a right moving wave)



Regular Waves

A right-moving (in the positive x-direction) wave is

$$\zeta(x, t) = a \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

where a is the wave amplitude, λ is the wavelength, and T is the wave period. Defining the wave number and circular frequency as $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$$\zeta(x, t) = a \cos(kx - \omega t)$$

This simple sinusoidal form is obtained under the following assumptions: Gravitational acceleration and dynamic wave pressure are the only factors balancing each other. Viscosity, surface tension, density variations and compressibility of water are all neglected. Hence potential theory is applicable and there exists a velocity potential.

Regular Waves

Velocity potential is given as

$$\phi(x, z, t) = \frac{ga}{\omega} \frac{\cosh k(h + z)}{\cosh kh} \sin(kx - \omega t)$$

where h is the water depth, and z is the vertical coordinate taken positive upwards. Particle velocity components and dynamic wave pressure are

$$u(x, z, t) = \frac{\partial \phi}{\partial x} = \frac{gka}{\omega} \frac{\cosh k(h + z)}{\cosh kh} \cos(kx - \omega t)$$

$$w(x, z, t) = \frac{\partial \phi}{\partial z} = \frac{gka}{\omega} \frac{\sinh k(h + z)}{\cosh kh} \sin(kx - \omega t)$$

$$p(x, z, t) = -\rho \frac{\partial \phi}{\partial t} = \rho ga \frac{\cosh k(h + z)}{\cosh kh} \cos(kx - \omega t)$$

Regular Waves

Dispersion relationship is given in various forms such as

$$\omega^2 = gk \tanh kh \quad \text{or} \quad c^2 = \frac{g}{k} \tanh kh$$

where c is the wave celerity: $c = \lambda/T = \omega/k$. For the case of deep water the preceding relationships reduce to

$$\phi(x, z, t) = \frac{\omega a}{k} e^{kz} \sin(kx - \omega t)$$

$$u(x, z, t) = \omega a e^{kz} \cos(kx - \omega t)$$

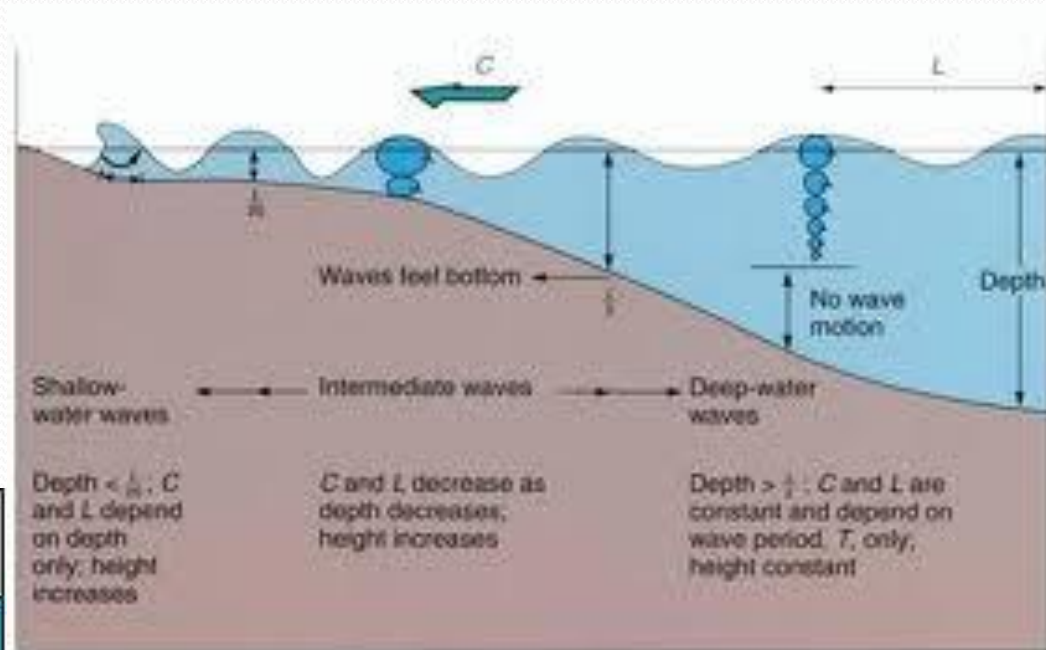
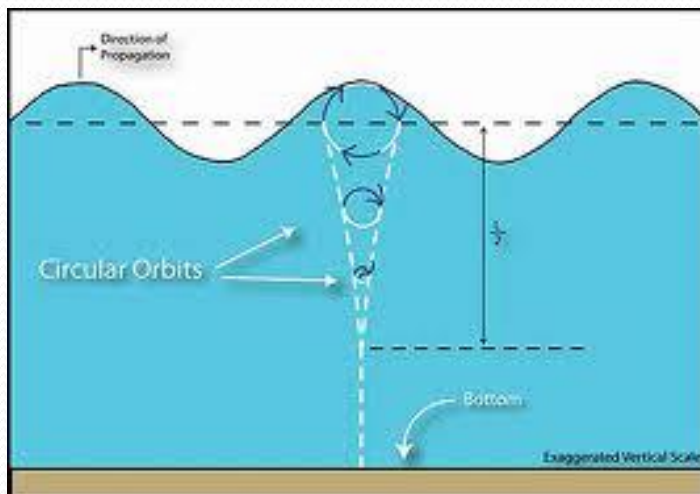
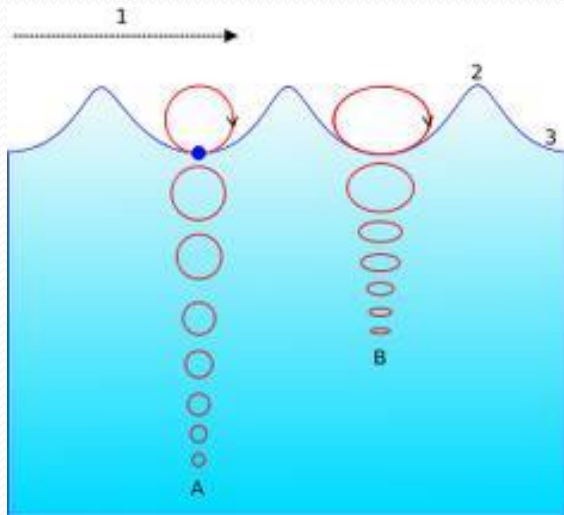
$$w(x, z, t) = \omega a e^{kz} \sin(kx - \omega t)$$

$$p(x, z, t) = \rho g a e^{kz} \cos(kx - \omega t)$$

$$\omega^2 = gk \quad \text{or} \quad c^2 = \frac{g}{k} \quad \text{or} \quad \lambda_0 = \frac{gT^2}{2\pi} = 1.56T^2$$

Regular Waves

Change in the form of orbital paths of sinusoidal waves



Regular Waves

- Example: A 5-sec wave with $H = 0.2 \text{ m}$ height is propagating in deep water. Find the velocity vector and dynamic pressure at a distance $x = 10 \text{ m}$ from the crest (the crest is located at $x = 0 \text{ m}$) at $z = -1 \text{ m}$ from the SWL, and time $t = 3 \text{ s}$.

- Solution: Lets begin by computing the wavelength λ_0 . For deep water waves $\lambda_0 = \frac{gT^2}{2\pi} = 1.56T^2$. If $T = 5 \text{ s}$, then $\lambda_0 = 39 \text{ m}$. Wave celerity $c_0 = \lambda_0/T = 39/5 = 7.8 \text{ m/s}$. Cyclic wave frequency and wave number are $\omega = 2\pi/T = 1.257 \text{ rad/s}$, $k = 2\pi/\lambda = 0.161 \text{ rad/m}$. The surface elevation is then

$$\zeta(x, t) = 0.1 \cos(0.161x - 1.257t) = 0.1 \cos[0.161(x - 7.8t)]$$

$$u(10, -1, 3) = 1.257 \cdot 0.1 \cdot e^{-1 \cdot 0.161} \cos(0.161 \cdot 10 - 1.257 \cdot 3) = -0.06 \text{ m/s}$$

$$w(10, -1, 3) = 1.257 \cdot 0.1 \cdot e^{-1 \cdot 0.161} \sin(0.161 \cdot 10 - 1.257 \cdot 3) = -0.09 \text{ m/s}$$

$$\vec{q} = u\vec{i} + w\vec{k} = -0.06\vec{i} - 0.09\vec{k} \text{ (m/s)}$$

$$p(10, -1, 3) = 1025 \cdot 9.81 \cdot 0.1 \cdot e^{-1 \cdot 0.161} \cos(0.161 \cdot 10 - 1.257 \cdot 3) = -476.4 \text{ Pa}$$

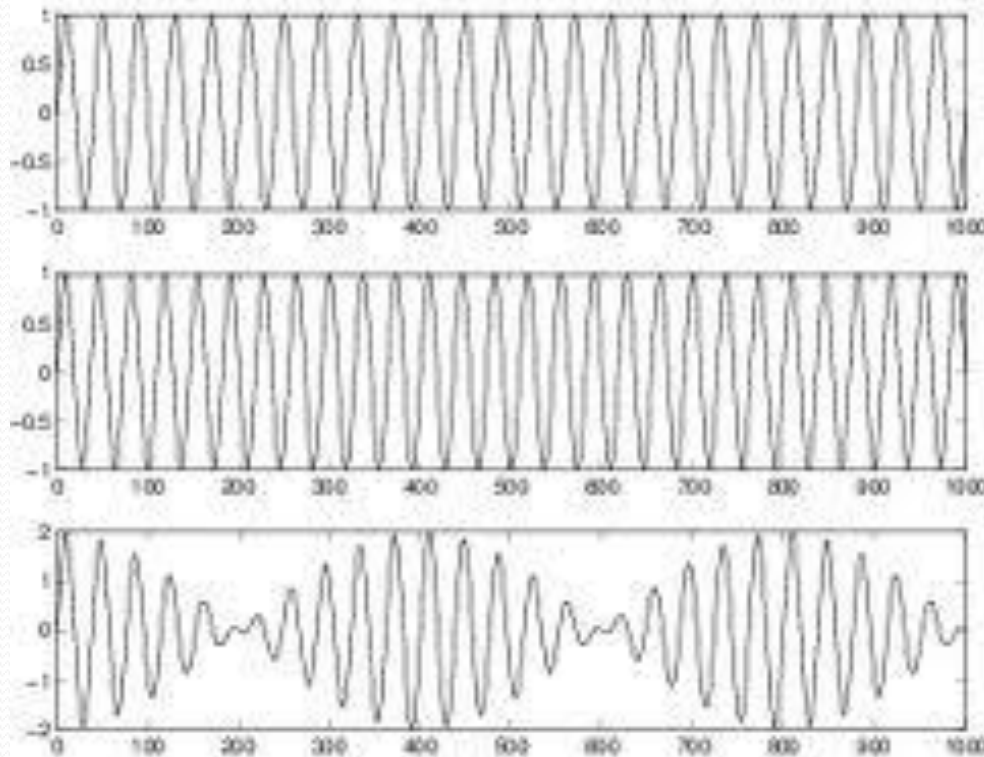
$$p_{Total} = p_{Hydr.} + p_{Dyn.} = -\rho g z + p_{Dyn.} = -1025 \cdot 9.81 \cdot (-1) - 476.4$$

$$p_{Total} = 9579 \text{ Pa} \cong 9.6 \text{ kPa}$$

Regular Waves

- Wave Groups

In sea waves are observed to travel in groups. By adding two sinusoidal waves with slightly different wavelengths and periods a wave group can be formed as seen below.



Regular Waves

- Group Velocity

From the simple construction of a wave group by adding two slightly different waves it can be shown that the group velocity may be computed from

$$c_g = \frac{d\omega}{dk} = \frac{1}{2} c \left(1 + \frac{2kh}{\sinh 2kh} \right)$$

Special forms of the group velocity for shallow and deep waters are respectively

$$c_g = c = \sqrt{gh} \quad \text{Shallow water}$$

$$c_g = \frac{1}{2} c_0 = \frac{1}{2} \sqrt{\frac{g}{k}} = 0.78 T \quad \text{Deep water}$$

Regular Waves

- Mean Wave Energy

Total mean wave energy per unit area is given by

$$E_T = E_K + E_P = \frac{1}{4}\rho g a^2 + \frac{1}{4}\rho g a^2 = \frac{1}{2}\rho g a^2 = \frac{1}{8}\rho g H^2$$

- Wave Energy Flux

In mechanical systems total energy is conserved. But for waves energy flux, which is defined below, is conserved.

$$P = \dot{E} = EC_g b = EC_g b_0 \cos \theta = \text{Constant}$$

where b is the distance between wave orthogonals, b_0 is an arbitrarily selected constant distance between orthogonals, and θ is the angle wave propagation direction makes with a reference zero angle direction.

Regular Waves

- Example: A tsunami is triggered in open ocean at a water depth of 3000 m. If the initial wave height of this tsunami is only 1 m compute its height at water depth 3 m. Compute its celerity in both cases. Assume unidirectional propagation.
- Solution: For unidirectional case ($\theta = 0^\circ$) the conservation of energy flux (wave power) gives

$$H^2 C_g = H^2 \sqrt{gh} = \text{Constant}$$

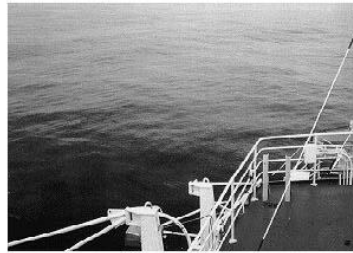
where use has been made of $C_g = \sqrt{gh}$ as tsunamis are shallow water waves for every water depth. (A tsunami has wavelength on the order of 100 km therefore $h/\lambda = 3/100 < 1/20$, which is the shallow water criterium.)

$$H_1^2 \sqrt{h_1} = H_2^2 \sqrt{h_2}$$
$$H_2 = H_1 \left(\frac{h_1}{h_2} \right)^{1/4} = 1 \cdot \left(\frac{4000}{4} \right)^{1/4} = 5.6 \text{ m}$$

$$c_1 = \sqrt{gh_1} = \sqrt{9.81 \cdot 3000} = 171.55 \text{ m/s} = 617.6 \text{ km/h}$$

$$c_2 = \sqrt{gh_2} = \sqrt{9.81 \cdot 3} = 5.4 \text{ m/s} = 19.5 \text{ km/h}$$

Sea Waves



Beaufort 1



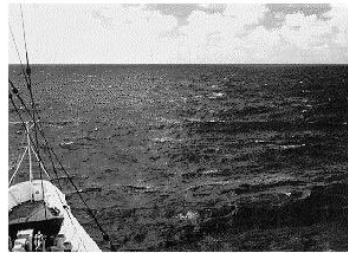
Beaufort 2



Beaufort 3



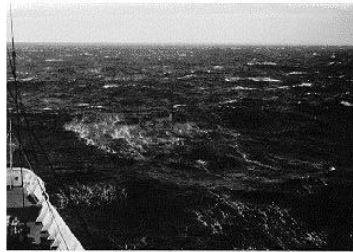
Beaufort 4



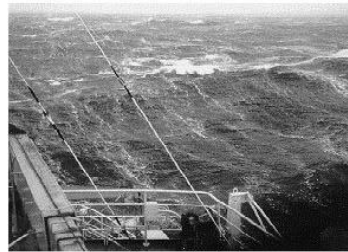
Beaufort 5



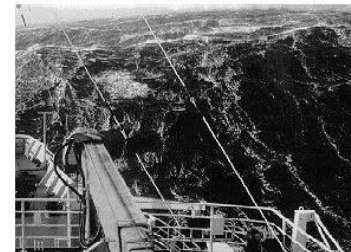
Beaufort 6



Beaufort 7



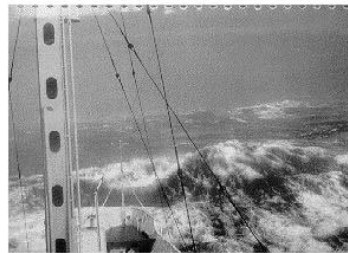
Beaufort 8



Beaufort 9



Beaufort 10



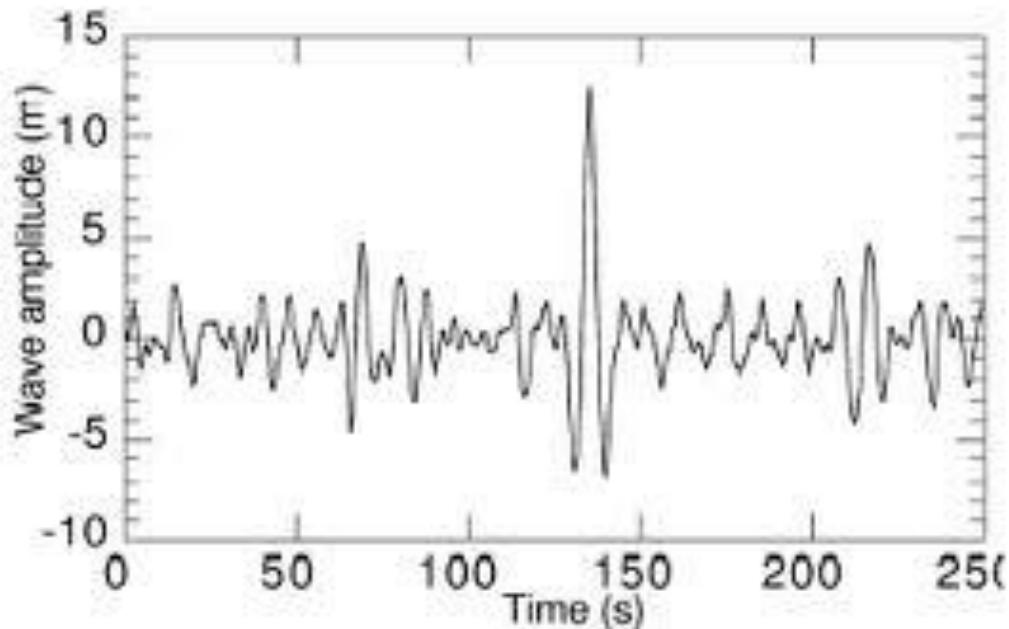
Beaufort 11



Beaufort 12

Sea Waves

- Wave Measurements: Most waves are recorded at a single fixed location. The simplest instrumentation can be used to simply record the water surface elevation as a function of time at that location. Since only a scalar, single point measurement is being made, the resulting record will yield no information about the direction of wave propagation.



Sea Waves

- The wave elevation (in the time domain) of a long-crested irregular sea, propagating along the positive x -axis, can be written as the sum of a large number of regular wave components (in the frequency domain):

$$\zeta(t) = \sum_{n=1}^N \zeta_{an} \cos(k_n x - \omega_n t + \varepsilon_n)$$

where, for each component,

ζ_{an} : wave amplitude component (m)

ω_n : circular frequency component (rad/s)

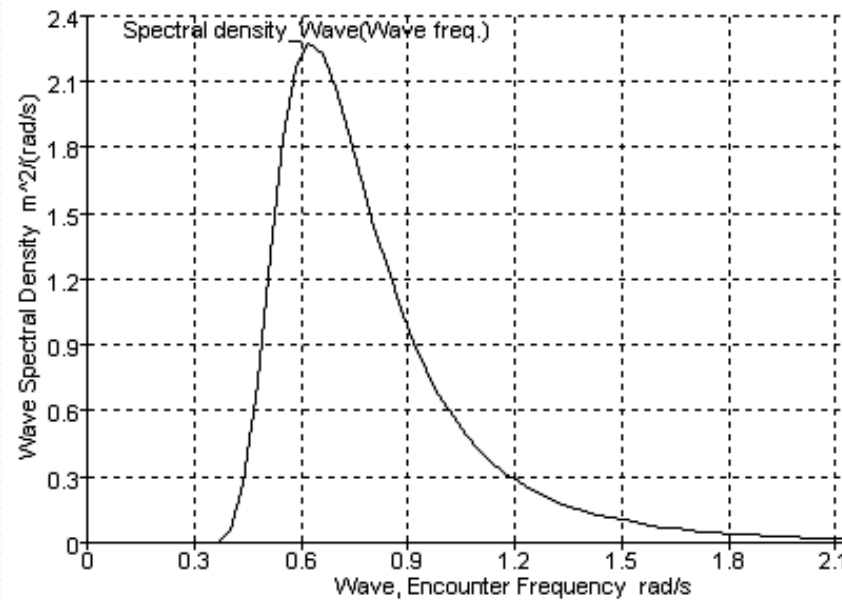
k_n : wave number component (rad/m)

ε_n : random phase angle component (rad)

Sea Waves

- Wave spectrum

Irregular waves are often described by a spectrum that indicates the amount of wave energy at different wave frequencies. A spectrum is shown by plotting spectral density against frequency; a typical wave spectrum is shown below:



These spectral representations of sea conditions are central to determining the response of a vessel in the seaway.

Sea Waves

- Wave Height Statistics

Successive wave heights are measured and classified in groups with intervals of, for instance, 0.5 meter. The number of wave heights in each group is then counted. These counts for each group are divided by the total number of wave heights to obtain the frequency quotients or the probability density function $\bar{p}(H)$. Finally these frequency quotients are added cumulatively to obtain the cumulative frequency quotients or the distribution function of wave heights $p(H)$. Specifically, the Rayleigh probability density function and the corresponding cumulative distribution function are given as follows.

Rayleigh P.D.F.
$$\bar{p}(H) = \frac{2H}{(H_{rms})^2} e^{-(H/H_{rms})^2}$$

Cumulative D.F.
$$p(H) = \int_0^H \bar{p}(H') dH' = 1 - e^{-(H/H_{rms})^2}$$

$p(H)$ indicates the probability that a given $H' \leq H$. Obviously the probability that $H' \geq H$ is $p^*(H) = 1 - p(H) = e^{-(H/H_{rms})^2}$.

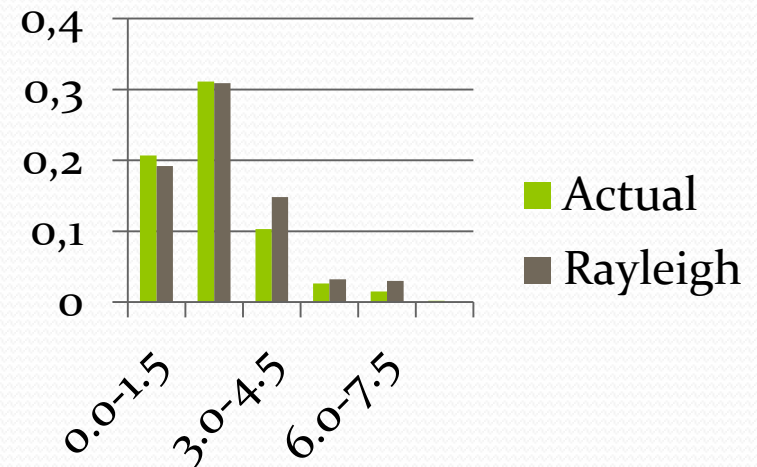
Sea Waves

- **Example:** The following wave heights were recorded in a fixed location in the ocean over a 12-hour period. $N = 4000 + 6000 + 2000 + 500 + 300 + 60 = 12860$

H (m)	0.0-1.5	1.5-3.0	3.0-4.5	4.5-6.0	6.0-7.5	7.5-10.5
Number of waves (n)	4000	6000	2000	500	300	60

Plot the PDF for the given data. Find the H_{rms} and plot the Rayleigh PDF.

H	n/N	ΔH	$\bar{p}_{Act.}(H')$	H'	$\bar{p}_{Ray.}(H')$
0.0-1.5	0.311	1.5	0.207	0.75	0.192
1.5-3.0	0.466	1.5	0.311	2.25	0.309
3.0-4.5	0.155	1.5	0.103	3.75	0.148
4.5-6.0	0.039	1.5	0.026	5.25	0.032
6.0-7.5	0.023	1.5	0.015	6.75	0.003
7.5-10.5	0.005	3.0	0.002	9.00	0.000



Sea Waves

- If $\hat{H}_{n/N}$ is the lowest of the highest n heights out of the total N heights then the average of the highest n heights out of the total N heights $H_{n/N}$ is computed by

$$H_{n/N} = \frac{\int_{\hat{H}_{n/N}}^{\infty} \bar{p}(H') H' dH'}{\int_{\hat{H}_{n/N}}^{\infty} \bar{p}(H') dH'}$$

Using the above moment formula with Rayleigh PDF the following relationships are obtained for various n/N values.

$$H_1 = H_{1/100} = 2.360 \cdot H_{rms}$$

$$H_{10} = H_{1/10} = 1.800 \cdot H_{rms}$$

$$H_s = H_{33} = H_{1/3} = 1.416 \cdot H_{rms}$$

$$\bar{H} = H_{100} = H_{1/1} = 0.886 \cdot H_{rms}$$

Sea Waves

- Relationship between H_s and the zeroth moment of the spectrum m_0 .

The zeroth moment of the wave spectrum is defined as

$$m_0 = \int_0^{\infty} S(\omega) d\omega = \frac{\text{Total Energy}}{\rho g \cdot \text{Area}}$$

However, it is also possible to write

$$m_0 = \frac{1}{N} \sum_{n=1}^N \frac{H_n^2}{8} = \frac{\text{Total Energy}}{\rho g \cdot \text{Area}}$$

From the definition of $H_{rms} = \sqrt{(\sum_{n=1}^N H_n^2)/N}$, writing $m_0 = H_{rms}^2 / 8$, and recalling $H_s = 1.416 H_{rms} \cong \sqrt{2} H_{rms}$ gives

$$H_s = 4\sqrt{m_0} = 4 \sqrt{\int_0^{\infty} S(\omega) d\omega}$$

Sea Waves

- Typical Wave Energy Spectra

It is often useful to define idealized wave spectra which represent the characteristics of real wave energy spectra.

Pierson-Moskowitz Spectrum: For fully developed seas, equilibrium spectrum.

$$S(\omega) = \frac{A}{\omega^5} e^{-(B/\omega^4)}$$

where $A = 8.11 \times 10^{-3} g^2$ and $B = 0.74(g/U_{wind})^4$ with $g = 9.81 \text{ m/s}^2$, U_{wind} is the nominal wind speed in m/s at a height of 19.5 m above the sea surface, and ω is the cyclic wave frequency in rad/s .

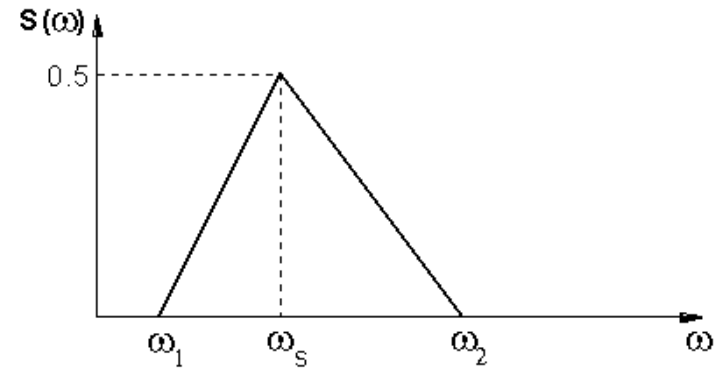
JONSWAP Spectrum: More suitable for fetch-limited coastal waters.

$$S(\omega) = \frac{A}{\omega^5} e^{-\frac{5}{4}(\omega_p/\omega)^4} \gamma^a$$

where A is the same as above and $a = e^{-\frac{1}{2}[(\omega - \omega_p)/\sigma\omega_p]^2}$ with ω_p being the peak frequency, $\sigma = 0.07$ if $\omega \leq \omega_p$, $\sigma = 0.09$ if $\omega > \omega_p$. Typically, $\gamma = 3.3$.

Sea Waves

- Example: A simplified wave energy spectrum is shown in the figure. Spectral energy is confined between $T_{min} = 4.2$ s and $T_{max} = 12.6$ s. It is also known that the wave components larger than the significant period contribute 1/3 of the total energy. Determine the significant wave height and period. The wave height H_d , which the total observed wave heights are greater for a month in a year.



First we compute $\omega_1 = 2\pi/T_{max} = 0.5$ rad/s, $\omega_2 = 2\pi/T_{min} = 1.5$ rad/s.

The zeroth moment $m_0 = \int_0^\infty S(\omega)d\omega = \frac{1}{2} \cdot 0.5 \cdot (\omega_2 - \omega_1) = 0.25$ m². $H_s = 4\sqrt{m_0} = 2$ m. It is given that the spectral area between ω_1 and ω_s is 1/3 of the total. Therefore $\frac{1}{2} \cdot 0.5 \cdot (\omega_s - \omega_1) = \frac{1}{3} \cdot 0.25$, $\omega_s = 0.833$ rad/s and $T_s = 2\pi/\omega_s = 7.54$ s. Finally, we want $p[H > H_d] = \frac{1}{12} = e^{-(H_d/H_{rms})^2}$. Since $H_s = \sqrt{2}H_{rms}$, $H_{rms} = \frac{2}{\sqrt{2}} = 1.414$ m. Then, $H_d = \sqrt{\ln 12} H_{rms} = 2.23$ m.

Sea Waves

- Hogging and Sagging of Ship Beam

(Smith, W. E. 'Hogging and Sagging Strains in a Seaway as Influenced by Wave Structure', TINA, 1883)

