SHIP THEORY - DYNAMICS

SEAKEEPING AND MANEUVERING

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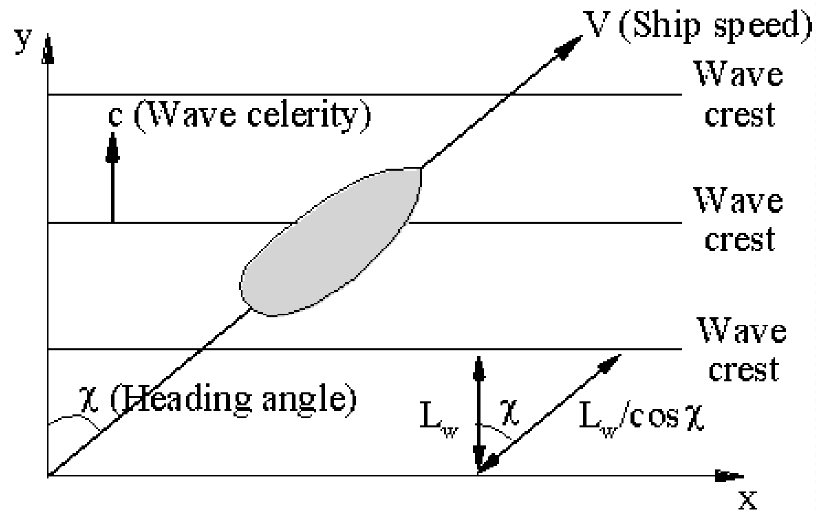
Frequency of Encounter

- A ship heading in a seaway does not experience the absolute wave frequency. Consider the figure below in which the ship moves with speed V at a heading angle χ to the wave crests.



The velocity component of ship in the direction of waves is $V\cos\chi$ and the relative speed of the ship with respect to the waves is $c - V\cos\chi$. Then, the time it takes the ship to travel from one crest to another is simply

$$T_e = \frac{L_w}{c - V\cos\chi} = \frac{cT_w}{c - V\cos\chi} = \frac{T_w}{1 - \frac{V}{c}\cos\chi}$$

where T_e is the encounter period, T_w the wave period, and c the wave celerity.

Frequency of Encounter

Accordingly, the frequency of encounter is

$$\omega_e = \omega\left(1 - \frac{V}{c}\cos\chi\right)$$

For deep water waves $\omega^2 = gk$ and since $c = \omega/k$, for deep water $c = g/\omega$.

$$\omega_e = \omega - \frac{\omega^2 V}{g}\cos\chi$$

If the ship is heading against the opposing waves $\chi > 90^0$ the sign is positive.

- **Example:** A 152.4 m-long-ship is moving in a regular train of waves at an angle of 40^0 to the wave crests. Every 15 s the bow meets a wave crest and it takes the wave 10 s to move from the bow to the stern. Find the ship speed.
- **Solution:** $L = 152.4\,m$ and $\chi = 90^0 - 40^0 = 50^0$. $t = \frac{L\cos\chi}{c - V\cos\chi} = 10\,s$ which gives $10(c - V\cos\chi) = 152.4\cos 50^0 = 97.96$. On the other hand for the encounter period we have $T_e = L_w/(c - V\cos\chi)$ therfore $L_w = 15 \cdot 9.796 = 146.99\,m$. For deep water waves $c = \sqrt{gL_w/2\pi}$ gives $c = 15.15\,m/s$. Finally, using $(c - V\cos\chi) = 9.796$ gives $V = 8.33\frac{m}{s} = 16.17\,knots$.

Uncoupled Heaving and Pitching

- **Uncoupled Heaving and Pitching of A Ship.** Similar to the heaving of a buoy in waves the uncoupled heaving and pitching of a ship in regular waves may be expressed as

$$(m+a)\ddot{z}+b\dot{z}+cz=F_a\cos(\omega_e t+\sigma_z)$$

$$(I_{yy}+A_{yy})\ddot{\theta}+B\dot{\theta}+C\theta=M_a\cos(\omega_e t+\sigma_\theta)$$

where F_a and M_a are respectively the amplitude and moment of wave force. According to the 'strip theory' the constants are defined as follows.

$$m=\int_L m_n dx\,,\qquad a=\int_L a_n dx\,,\qquad b=\int_L \left(b_n-V\frac{da_n}{dx}\right)dx$$

$$c=\int_L c_n dx=\rho g A_w,\qquad I_{yy}=\int_L m_n x^2 dx\,,\qquad A_{yy}=\int_L a_n x^2 dx$$

$$B=\int_L \left(b_n-V\frac{da_n}{dx}\right)x^2 dx\,,\qquad C=\int_L c_n x^2 dx+V\cdot E$$

$$E=-\int_L \left(b_n-V\frac{da_n}{dx}\right)x dx$$

Uncoupled Heaving and Pitching

- **Solution of unforced pitching.** The equation for unforced pitching

$$\ddot{\theta} + \left(\frac{B}{I_{yy} + A_{yy}}\right)\dot{\theta} + \left(\frac{C}{I_{yy} + A_{yy}}\right)\theta = 0$$

Defining

$$n_\theta = \frac{B}{2(I_{yy} + A_{yy})}, \qquad {\omega_{n\theta}}^2 = \frac{C}{(I_{yy} + A_{yy})}$$

The solution reads

$$\theta(t) = e^{-n_\theta t}\left[\theta_0 \cos(\omega_{d\theta}' \ t) + \frac{\dot{\theta}_0 + n_\theta \theta_0}{\omega_{n\theta}'}\sin(\omega_{d\theta}' \ t)\right]$$

where $\omega_{d\theta}'$ is the damped frequency of pitching. The natural frequency may be re-expressed as follows. If $V = 0$ then $C = \rho g I_L$ and since $I_L/\nabla = \overline{BM}_L$, $C = \rho g \nabla \overline{BM}_L$. Letting $I_{yy} = m{k_{yy}}^2 = \rho g \nabla {k_{yy}}^2$ and $e_\theta = A_{yy}/I_{yy}$ gives

$$\omega_{n\theta} = \sqrt{\frac{g\overline{BM}_L}{{k_{yy}}^2 (1 + e_\theta)}}, \qquad T_{n\theta} = 2\pi k_{yy}\sqrt{\frac{1 + e_\theta}{g\overline{GM}_L}}$$

Uncoupled Heaving and Pitching

- **Solution of forced pitching.** The equation for unforced pitching

$$\ddot{\theta} + \left(\frac{B}{I_{yy} + A_{yy}}\right)\dot{\theta} + \left(\frac{C}{I_{yy} + A_{yy}}\right)\theta = M_a \cos(\omega_e t + \sigma_\theta)$$

The particular solution is

$$\theta = \theta_a \cos(\omega_e t + \varepsilon_\theta)\,, \qquad \theta_a = \zeta_a \frac{M_a/C}{\sqrt{\left(1 - {\Lambda_\theta}^2\right)^2 + {\kappa_\theta}^2\,{\Lambda_\theta}^2}}$$

where $\Lambda_\theta = \omega_e/\omega_{n\theta},\ \kappa_\theta = B/\sqrt{C(I_{yy} + A_{yy})},\ \varepsilon_\theta = \sigma_\theta - \tau_\theta, \tau_\theta = \tan^{-1}[B\omega_e/$