SHIP THEORY - PROPULSION

PROPELLER DESIGN AND CAVITATION

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Dimensional Analysis

- The thrust T of a propeller could depend upon:

a) Mass density of water, ρ.
b) Size of the propeller, represented by diameter D.
c) Speed of advance, V_A.
d) Acceleration due to gravity, g.
e) Speed of rotation, n.
f) Pressure in the fluid, p.
g) Viscosity of the water, μ.

$$T = f(\rho^a, D^b, {V_A}^c, g^d, n^e, p^f, \mu^g)$$

$$\left(\frac{ML}{T^2}\right)^1 = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{L}{T}\right)^c \left(\frac{L}{T^2}\right)^d \left(\frac{1}{T}\right)^e \left(\frac{M}{LT^2}\right)^f \left(\frac{M}{LT}\right)^g$$

Dimensional Analysis

- Multiplying and equating the powers

$$a = 1 - f - g$$
$$b = 1 + 3a - c - d + f + g$$
$$c = 2 - 2d - e - 2f - g$$

Substituting a and c in the expression for b gives $b = 2 + d + e - g$ then

$$T = \rho D^2 {V_A}^2 \left(\frac{gD}{{V_A}^2}\right)^d \left(\frac{nD}{V_A}\right)^e \left(\frac{p}{\rho {V_A}^2}\right)^f \left(\frac{\nu}{V_A D}\right)^g$$

where $\nu = \mu/\rho$ is the kinematic viscosity. In general we can write

$$\frac{T}{\frac{1}{2}\rho D^2 {V_A}^2} = f\left[\frac{gD}{{V_A}^2}, \frac{nD}{V_A}, \frac{p}{\rho {V_A}^2}, \frac{\nu}{V_A D}\right]$$

Note that since the disc area of the propeller , $A_0 = (\pi/4)D^2$, is proportional to D^2, the thrust coefficient can also be written in the form

$$\frac{T}{\frac{1}{2}\rho A_0 {V_A}^2}$$

Dimensional Analysis

- If the model and ship quantities are denoted by the suffixes M and S, respectively, and if λ is the linear scale ratio, then

$$D_S/D_M = \lambda$$

If the propeller is run at the correct Froude speed of advance, $Fr_S = Fr_M$,

$$\frac{V_{AS}}{V_{AM}} = \lambda^{1/2}$$

The slip ratio has been defined as $(1 - V_A/Pn)$. For geometrically similar propellers, therefore, the nondimensional quantity nD/V_A must be the same for model and ship. Thus, as long as $gD/{V_A}^2$ and nD/V_A are the same in ship and model

$$T \propto D^2 {V_A}^2$$

$$\frac{D_S}{D_M} = \lambda, \quad \frac{V_{AS}}{V_{AM}} = \lambda^{1/2}, \quad \frac{T_S}{T_M} = \frac{{D_S}^2}{{D_M}^2} \cdot \frac{{V_{AS}}^2}{{V_{AM}}^2} = \lambda^3$$

$$\frac{n_S D_S}{V_{AS}} = \frac{n_M D_M}{V_{AM}}, \quad \frac{n_S}{n_M} = \frac{D_M}{D_S} \cdot \frac{V_{AS}}{V_{AM}} = \frac{1}{\lambda} \cdot \lambda^{1/2} = \frac{1}{\lambda^{1/2}}, \quad n_M = n_S \cdot \lambda^{1/2}$$

Dimensional Analysis

- The thrust power is given by $P_T = T \cdot V_A$, so that

$$\frac{P_{TS}}{P_{TM}} = \frac{T_S}{T_M} \cdot \frac{V_{AS}}{V_{AM}} = \lambda^{3.5}, \qquad \frac{Q_S}{Q_M} = \frac{P_{DS}}{n_S} \cdot \frac{n_M}{P_{DM}} = \lambda^{3.5} \cdot \lambda^{0.5} = \lambda^4$$

If the model results were plotted as values of

$$C_T = \frac{T}{\frac{1}{2}\rho D^2 {V_A}^2}, \qquad C_Q = \frac{Q}{\frac{1}{2}\rho D^3 {V_A}^2}$$

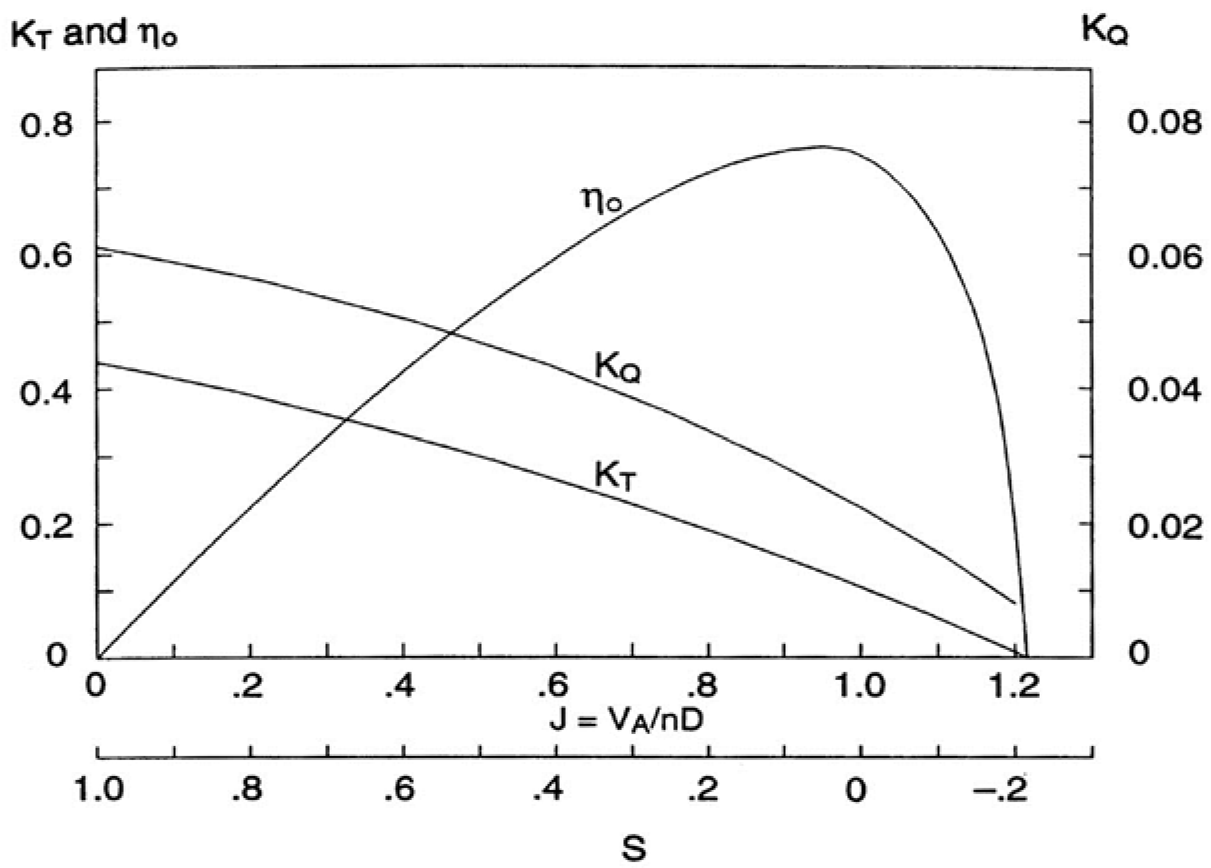
to a base of V_A/nD or J, therefore, the values would be directly applicable to the ship. But the above coefficients have the disadvantage that they become infinite for zero speed of advance, a condition sometimes occurring in practice. Since J or V_A/nD is the same for model and ship, V_A may be replaced by nD.

$$K_T = \frac{T}{\rho n^2 D^4}, \qquad K_Q = \frac{Q}{\rho n^2 D^5}$$

$$J = \frac{V_A}{nD}, \qquad \eta_0 = \frac{P_T}{P_{D_0}} = \frac{TV_A}{2\pi n Q_0} = \frac{J}{2\pi} \cdot \frac{K_T}{K_{Q_0}}, \qquad \sigma = \frac{p_0 - p_v}{\rho n^2 D^5}$$

where $P_T = T \cdot V_A$ is the thrust power and $P_{D_0} = 2\pi n \cdot Q_0$ is the delivered power of the open water propeller.

Dimensional Analysis



Propeller-Hull Interaction

- **Wake:** Previously we have considered a propeller working in *open water*. When a propeller operates behind the model or ship hull the conditions are considerably modified. The propeller works in water which is disturbed by the passage of the hull, and in general the water around the stern acquires a forward motion in the same direction as the ship. This forward-moving water is called the wake, and one of the results is that the propeller is no longer advancing relatively to the water at the same speed as the ship, V, but at some lower speed V_A, called the speed of advance.

Propeller-Hull Interaction

- Froude wake fraction

$$w_F = \frac{V - V_A}{V_A}, \qquad V_A = \frac{V}{1 + w_F}$$

- Taylor wake fraction

$$w = \frac{V - V_A}{V}, \qquad V_A = V(1 - w)$$

- Resistance augment fraction and thrust deduction fraction

$$a = \frac{T - R_T}{R_T} = \frac{T}{R_T} - 1, \qquad T = (1 + a)R_T$$

$$t = \frac{T - R_T}{T} = 1 - \frac{R_T}{T}, \qquad R_T = (1 - t)T$$

Power Definitions

- **Brake power** is usually measured directly at the crankshaft coupling by means of a torsion meter or dynamometer. It is determined by a shop test and is calculated by the formula $P_B = (2\pi n)Q_B$, where n is the rotation rate, revolutions per second and Q_B is the brake torque, $N \cdot m$.

- **Shaft power** is the power transmitted through the shaft to the propeller. For diesel-driven ships, the shaft power will be equal to the brake power for direct-connect engines (generally the low-speed diesel engines). For geared diesel engines (medium- or high-speed engines), the shaft horsepower will be lower than the brake power because of reduction gear "losses." Shaft power is usually measured aboard ship as close to the propeller as possible by means of a torsion meter. The shaft power is given by $P_S = (2\pi n)Q_S$.

Power Definitions

- **Delivered power** is the power actually delivered to the propeller. There is some power lost in the stern tube bearing and in any shaft tunnel bearings between the stern tube and the site of the torsion meter. The power actually delivered to the propeller is therefore somewhat less than that measured by the torsion meter. This delivered power is given the symbol P_D.
- **Thrust power** is the power delivered by propeller as it advances through the water at a speed of advance V_A, delivering the thrust T. The thrust power is $P_T = TV_A$.
- **Effective power** is defined as the resistance of the hull, R, times the ship speed V, $P_E = RV$.
- **Propulsive efficiency** is defined as

$$\eta_D = \frac{P_E}{P_D} = \frac{P_E}{P_T}\frac{P_T}{P_{D_0}}\frac{P_{D_0}}{P_D} = \frac{RV}{TV_A}\eta_0\eta_R = \frac{T(1-t)V}{TV(1-w)}\eta_0\eta_R = \frac{(1-t)}{(1-w)}\eta_0\eta_R$$

$$\eta_D = \eta_H\eta_0\eta_R, \qquad \eta_H = \frac{(1-t)}{(1-w)}, \qquad \eta_R = \frac{P_{D_0}}{P_D}$$

Different Design Approaches

- **Case 1:** P_D, RPM, V_A are known and $D_{opt.}$ is required.

In this case the unknown parameter D_{opt} may be eliminated from the diagrams by plotting K_Q/J^5 versus J instead of K_Q versus J as follows.

$$\frac{K_Q}{J^5} = \left(\frac{Q}{\rho n^2 D^5}\right)\left(\frac{nD}{V_A}\right)^5 = \frac{Qn^3}{\rho {V_A}^5} = \frac{(2\pi Q n)n^2}{2\pi\rho {V_A}^5} = \frac{P_D n^2}{2\pi\rho {V_A}^5}$$

$${K_Q}^{1/4} J^{-5/4} = \left[\frac{Qn^3}{\rho {V_A}^5}\right]^{1/4} = 0.1739\sqrt{B_{p_1}}$$

which are the variables used in charts between pages 192-196. On these charts the optimum η_0 and $1/J$ are read off at the intersection of known ${K_Q}^{1/4}J^{-5/4}$ value. Afterwards D_{opt} is computed using the optimum $1/J$ value as $D_{opt} = V_A/nJ$.

Different Design Approaches

- **Case 2:** P_D, D, V_A are known and RPM is required.

In this case the unknown parameter n may be eliminated from the diagrams by plotting K_Q/J^3 versus J instead of K_Q versus J as follows.

$$\frac{K_Q}{J^3} = \left(\frac{Q}{\rho n^2 D^5}\right)\left(\frac{nD}{V_A}\right)^3 = \frac{Qn}{\rho D^2 {V_A}^3} = \frac{2\pi Qn}{2\pi\rho D^2 {V_A}^3} = \frac{P_D}{2\pi\rho D^2 {V_A}^3}$$

$$ {K_Q}^{1/4} J^{-3/4} = \left[\frac{Qn}{\rho D^2 {V_A}^3}\right]^{1/4} = 1.75\sqrt{B_{p_2}}$$

which are the variables used in charts between pages 197-201. On these charts the optimum η_0 and $1/J$ are read off at the intersection of known ${K_Q}^{1/4}J^{-3/4}$ value. Afterwards n is computed using the optimum $1/J$ value as $n = V_A/JD$.

Typical Propeller Design

- Given Data

Service speed: $V = 21\ knots = 10.8\ m/s$

Effective power with model-ship correlation allowance: $P_E = 9592\ kW$

Estimated propulsive efficiency: $\eta_D = P_E/P_D = 0.75$

Immersion depth of propulsion shaft: $h = 7.5\ m$

Estimated delivered power at 21 $knots$: $P_D = P_E/\eta_D = 9592/0.75 = 12789\ kW$

Empirically determined wake fraction: $w = 0.20$

Empirically determined thrust deduction fraction: $t = 0.15$

Estimated relative rotative efficiency: $\eta_R = 1.05$

- Design Calculation

Selected propeller diameter (with adequate clearance): $D = 6.4\ m$

Selected number of blades (from consideration of vibration forces): $Z = 4$

Calculation of velocity of advance (wake speed): $V_A = V(1 - w) = 10.8(1 -$

Typical Propeller Design

Wageningen B-Series charts: ${K_Q}^{1/4} \cdot J^{-3/4} = \left[\frac{P_D}{2\pi\rho D^2 {V_A}^3}\right]^{1/4} =$

$\left[\frac{12477}{2\pi \cdot 1.025 \cdot (6.4)^2 \cdot (8.64)^3}\right]^{1/4} = 0.52039$

Using the charts given on pages 197 (B4-40), 198 (B4-55), and 199 (B4-70) for ${K_Q}^{1/4} \cdot J^{-3/4} = 0.52039$ from the optimum propeller efficiency curve we get

Expanded blade area ratio:	0.40	0.55	0.70
$1/J$ at optimum efficiency:	1.260	1.275	1.290
$RPM = 60 \cdot (V_A/D) \cdot (1/J)$:	102.1	103.3	104.5
Corresponding P/D:	1.110	1.090	1.070
Open water efficiency η_0:	0.673	0.670	0.663

We proceed to choose the blade area ratio by applying a cavitation criterion given by Keller.

Typical Propeller Design

The required thrust is

$$T = \frac{R}{1-t} = \frac{RV}{(1-t)V} = \frac{P_E}{(1-t)V} = \frac{9592}{(1-0.15)\cdot 10.8} = 1044.9\ kN$$

The Keller area criterion for a single-screw vessel gives

$$\frac{A_E}{A_0} = \frac{(1.3+0.3\cdot Z)\cdot T}{(P_0-P_v)\cdot D^2} + k$$

where $P_0 - P_v = P_{atm.} - P_{vapour} + \rho g h = 98100 - 1750 + 1025\cdot 9.81\cdot 7.5 = 171764\ N/m^2 = 171.8\ kN/m^2$, and k is a constant varying from 0 to 0.2. Taking $k = 0.2$,

$$\frac{A_E}{A_0} = \frac{(1.3+0.3\cdot 4)\cdot 1044.9}{171.8\cdot (6.4)^2} + 0.2 = 0.571$$

Interpolating between $A_E/A_0 = 0.55$ and $A_E/A_0 = 0.70$ for $A_E/A_0 = 0.571$ we get $N = 103.5\ rpm$, $P/D = 1.087$, and $\eta_0 = 0.669$. Accordingly the η_D value becomes:

$$\eta_D = \frac{(1-t)}{(1-w)}\cdot\eta_0\cdot\eta_R = \frac{(1-0.15)}{(1-0.20)}\cdot 0.669\cdot 1.05 = 0.746$$

This compares well with the value of 0.750 assumed. If a larger difference had been found, a new estimate of the power would have to be made, using $\eta_D = 0.746$.

Typical Propeller Design

- Another Example of Calculating Optimum RPM

A somewhat different example of calculating optimum RPM is given below. A common problem for the propeller designer is the design of a propeller when the required propeller thrust T (e.g. from a model test equal to the resistance corrected for the thrust deduction) and the propeller diameter D (e.g. from the available clearance) are known. The advance velocity of the propeller may be estimated from the ship speed and wake fraction. The unknowns are the required power and the engine RPM. Suppose that the following data is given for a container vessel.

Thrust: $T = 142\ tons = 142000 \cdot 9.81\ N = 1393000\ N = 1393\ kN$

Ship speed: $V = 21\ knots = 10.8\ m/s$

Advance velocity (Wake speed): $V_A = 21(1 - 0.2) = 16.8\ knots = 8.64\ m/s$

Selected propeller diameter (with adequate clearance): $D = 7.0\ m$

Selected number of blades (from consideration of vibration forces): $Z = 4$

Immersion depth of propulsion shaft: $h = 5.0\ m$

- Design Calculation

We begin by considering Keller's formula for AER with $k = 0$ for the given ship

$$EAR = \frac{A_E}{A_0} = \frac{(1.3 + 0.3 \cdot 4) \cdot 1393}{(98.1 - 1.75 + 1.025 \cdot 9.81 \cdot 5.0) \cdot (7.0)^2} = 0.485$$

For single skrew wake field $k = 0.2$, it may be less if the wake field is good.

Typical Propeller Design

Obviously k is an important parameter affecting the value of AER. If it were selected as 0.2, EAR would be 0.685 instead of 0.485. Now that we have taken $k = 0$ and consequently the minimum EAR must be 0.485. To be on the safe side let us select $EAR = 0.55$. Already we have selected $Z = 4$ therefore our propeller now is B4-55. Consequently we can use the $K_T - K_Q - J$ diagram of B4-55 series. We know the thrust T and the diameter D, hence seek for the RPM. To use the diagram we need to know both K_T and J; however at present we can only calculate K_T/J^2 as follows

$$\frac{K_T}{J^2} = \frac{T}{\rho V^2 D^2} = \frac{1393000}{1025 \cdot 8.65^2 \cdot 7^2} = 0.3707$$

Now when we select a $J - P/D$ pair in the diagram it must be such that $K_T/J^2 = 0.3707$. This may be done either by trial-and-error followed by linear iteration or by the graphical method. The graphical method gives the result directly but in order to use it we must draw the curve corresponding to $K_T = 0.3707J^2$ for a range of J values. Wherever this curve crosses a K_T value in the diagram of B4-55, that particular $K_T - J$ pair corresponding to a definite P/D ratio in the diagram is a correct $K_T - J$ pair. Supposing that we have established the following table for various P/D ratios using the $K_T - K_Q - J$ diagram (B4-55)

Typical Propeller Design

P/D	J	K_T	K_Q	η_0	K_T/J^2
0.8	0.594	0.131	0.0195	0.636	0.371
0.9	0.648	0.156	0.0248	0.648	0.371
1.0	0.699	0.181	0.0310	0.651	0.371
1.1	0.747	0.207	0.0380	0.649	0.371
1.05	0.723	0.194	0.0344	0.650	0.371
0.95	0.674	0.168	0.0278	0.650	0.371

Our aim now is to select the propeller with maximum open-water efficiency. The first examination indicates it may be $P/D = 1$ but to make sure we try the close neigborhood of it; $P/D = 0.95$ and $P/D = 1.05$. From these results we see that $P/D = 1$ really gives the maximum efficiency hence we select it. Thus our final decision for the propeller is B4-55, $P/D = 1$, $\eta_0 = 0.651$, $J = 0.699$ so that $n = \frac{V_A}{JD} = \frac{8.65}{0.699 \cdot 7} = 1.768\, rps$, $N = 60 \cdot n = 106\, rpm$. Finally, since the torque coefficient $K_Q = 0.0310$, $Q = \rho n^2 D^5 K_Q = 1025 \cdot 1.768^2 \cdot 7^5 \cdot 0.0310$, $Q = 1669322\, N \cdot m$. Power to be delivered $P_D = 2\pi n Q/1000 = 18544\, kW$

Typical Propeller Design

- Example of Calculating Optimum Diameter

In this example we suppose that from the beginning we have selected the engine and its RPM hence we are to select a diameter for maximum efficiency.

For a fast patrol boat the following data is given.

- Advance velocity of propeller: $V_A = 28\ knots = 14.4\ m/s$
- Delivered power : $P_D = 440\ kW$
- $RPM = 720,\ n = 720/60 = 12\ rps$
- $Z = 5$
- $EAR = A_E/A_0 = 0.75$

Typical Propeller Design

To eliminate the unknown diameter we must use K_Q/J^5 as the parameter.

$$\frac{K_Q}{J^5} = \left(\frac{Q}{\rho n^2 D^5}\right)\left(\frac{nD}{V_A}\right)^5 = \frac{Qn^3}{\rho V_A{}^5} = \frac{2\pi Qn\cdot n^2}{2\pi\rho V_A{}^5} = \frac{P_D n^2}{2\pi\rho V_A{}^5} = \frac{440000\cdot 12^2}{2\pi\cdot 1025\cdot 14.4^5} = 0.016$$

Applying any one of the approaches (graphical or interpolative) described in the previous problem we can establish the following table.

P/D	J	K_T	K_Q	η_0	K_Q/J^5
1.200	1.085	0.098	0.0237	0.713	0.016
1.300	1.140	0.124	0.0306	0.735	0.016
1.400	1.190	0.153	0.0387	0.747	0.016
1.350	1.165	0.138	0.0346	0.742	0.016

In this case the most efficient choice is $P/D = 1.4$ and $J = 1.19$ so that $D = V_A/nJ$, $D = 14.4/(12 \cdot 1.19) = 1.01\ m$, $D = 1\ m$. Since $K_T = 0.153$, $T = \rho n^2 D^4 K_T$, $T = 23500\ N$. If this thrust is not in accord with the resistance calculations of the boat calculations must be repeated for a different speed till the convergence is gained. Finally the cavitation possibility must be checked at least by using the Keller formula. In this case Keller formula gives minimum $EAR = 0.59$ hence OK.