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Applications of A Generalized Kadomtsev-Petviashvili Type Equation for Varying Water Depths

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Abstract. Applications of a Kadomtsev-Petviashvili type equation with generalized dispersion and shoaling terms are presented. A finite-difference scheme is adopted for the numerical solution of the new wave equation to explore the extended range of application areas. Several experimental cases are simulated and compared with the measurements. Overall performance of the new equation is quite satisfactory.

Keywords: Kadomtsev-Petviashvili equation, improved dispersion, improved shoaling, varying water depths.
PACS: 47.35.Bb; 47.35.Fg.

INTRODUCTION

Kadomtsev and Petviashvili derived a nonlinear wave equation [1] which might be interpreted as the directional version of Korteweg and de Vries equation [2]. The origin of the Kadomtsev and Petviashvili equation, briefly the KP equation, may be traced back to the wave equation of Boussinesq [3], which is a weakly nonlinear weakly dispersive wave model. The present work explores the performance of a KP type equation with relatively better dispersion characteristics, based on a previous KdV type equation for varying depths [4]. The numerical treatment of the new equation is done by a Crank-Nicolson type finite-difference formulation similar to Feng and Mitsui [5]. Starting with examination of shoaling properties of the equation for varying bathymetry, simulation of a test case is presented. Compared to the existing KP type models the newly introduced model performs remarkably better for relatively shorter waves over varying depths. These improved aspects are quite important for realistic simulations of waves propagating in nearshore regions.

GENERALIZED KP TYPE EQUATION

A new KP type equation with mixed dispersion and additional linear shoaling terms is given in [6] as an extension of the KdV type equation [4].

$$\zeta_{xt} + C\zeta_{xx} - pCh^2\zeta_{xxxx} - qh^2\zeta_{xxxt} + \frac{3C}{4h}(\zeta^2)_{xx} + \frac{3C}{4h}h_x\zeta_x - r_p Chh_x\zeta_{xxx} - s_q hh_x\zeta_{xxt} = -\frac{1}{2}C\zeta_{yy} \tag{1}$$

where ζ is the free surface displacement as measured from the still water level, h is the water depth and $C = \sqrt{gh}$ is defined as the non-dispersive phase velocity with g denoting the gravitational acceleration. The non-dimensional coefficients $p = (1+2\beta)/6$, $q = (1+\beta)/3$, $r = (15+32\beta)/24$, $s = 5(1+\beta)/6$ are employed for the sake of a simpler notation with $r_p = r + 5p/2$ and $s_q = s + 2q$. β stands for a non-dimensional dispersion parameter while subscripts denote partial differentiation with respect to the indicated variable. In the derivation of Eq.(1) the second and higher derivatives of the water depth h are neglected in accord with the mild-slope assumption.

The above generalized KP type equation embodies all the known KP type equations as special cases: for constant depth setting $\beta = -1$ ($p = -1/6$, $q = 0$) gives the original KP equation while $\beta = -1/2$ ($p = 0$, $q = 1/6$) gives the KP equation corresponding to the KdV model of Benjamin et al. [7], the so-called BBM model. Moreover, the equation contains terms accounting for depth variations so that it can model changes in wave amplitude due to changes in water depth. Accuracy and consistency of shoaling characteristics of the unidirectional version; that is to say, the corresponding KdV equation has already been investigated analytically and shown that shoaling properties of the model equation are in exact agreement with those obtained from the constancy of energy flux [4].

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The dispersion relationship of Eq.(1) may be obtained as

$$\frac{\mathbf{C}_{KP}}{C} = \left(\frac{1+\frac{1}{6}(1+2\beta)k_x^2h^2 + \frac{1}{2}(k_y^2/k_x^2)}{1+\frac{1}{3}(1+\beta)k_x^2h^2} \right) \left(\frac{k_x}{k^2} \right) \mathbf{k} \tag{2}$$

where $C = \sqrt{gh}$ as defined before and $\mathbf{C}_{KP}$ denotes the phase velocity vector of the new KP type equation. For selecting the dispersion parameter β the main propagation direction; namely, the wave propagation in the $x-$direction is considered. For unidirectional waves $\theta = 0$ then $k_x = k$ and $k_y = 0$, which renders the above dispersion relationship identical with the improved or generalized KdV equation presented in [4]. Without repeating the arguments and analysis presented in detail in [4] there are basically two prominent choices for β: $-1/20$ or 0. If the dispersion relationship is to correspond to the fourth-order Padé approximation of the exact relationship, $\beta = -1/20$ is the choice; on the other hand, if the shoaling characteristics of the equation is to be in perfect agreement with the energy flux concept, $\beta = 0$ should be preferred.

The exact dispersion relationship of linear theory is manipulated such that it is expressed as a function of k_xh just like the KP dispersion relationship given in Eq.(2):

$$\frac{\mathbf{C}_{EX}}{C} = \sqrt{\frac{\tanh\left(k_xh\sqrt{1+(k_y/k_x)^2}\right)}{k_xh\sqrt{1+(k_y/k_x)^2}}} \frac{\mathbf{k}}{\sqrt{k_x^2+k_y^2}} \tag{3}$$

Figure 1 compares $|\ \mathbf{C}_{EX}/C\ |$ of linear theory with $|\ \mathbf{C}_{KP}/C\ |$ for $\beta = -1/20$ for two different direction angles $\theta = 0^o$ and $\theta = 30^o$. k_y/k_x is assigned to $\tan\theta$ while k_xh is varied over the range $0-\pi$. Even for a directional propagation angle as large as $\theta = 30^o$ dispersion characteristics deteriorate quite slightly for the range of wave numbers approaching the theoretical limit of deep water waves $kh = \pi$.

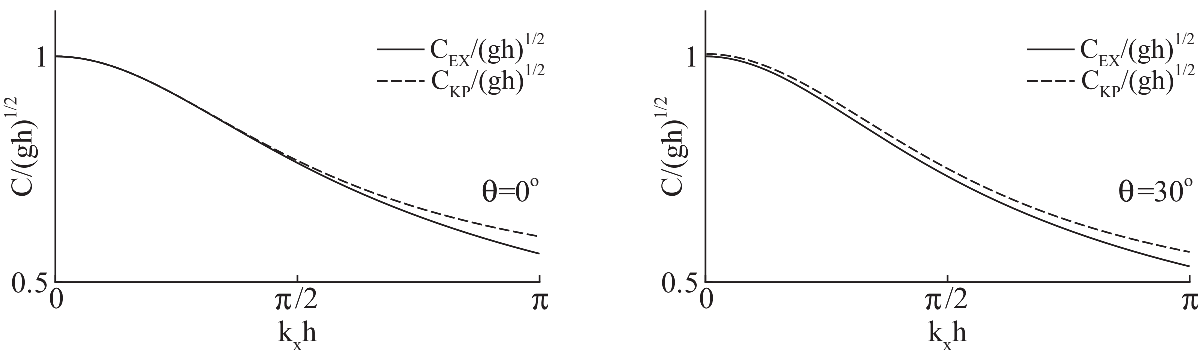

FIGURE 1. Linear dispersion relationship of the new KP type equation for directional propagation compared with linear theory for two different propagation angles $\theta = 0^o$ and $\theta = 30^o$.

A NUMERICAL SIMULATION

The case considered here is the experimental measurements of Whalin [8] for waves converging over a bottom topography that acts as a focusing lens. The physical wave tank used in the experiments was 84 ft = 25.6 m long and 20 ft = 6.096 m wide. In the middle part of the tank eleven semicircular steps were evenly spaced to form a topographical lens. Experiments were carried out by generating regular waves with periods $T = 1, 2,$ and 3 s. Figure 2 shows the perspective view of the fully-developed numerical wave tank for $T = 2$ s period incident waves. The simulation was performed with a span-wise resolution Δy of $1/12$ of the wave tank width. For incident wave period $T =$ 2 s the incident wave amplitude is and $a_0 = 0.75$ cm in water depth of $h_0 = 0.4572$ m. The time-step and the $x-$direction resolution were respectively $\Delta t = T/50$ s and $\Delta x = L_m/50$ m with L_m denoting the mean wavelength computed as the average of the deep-water $h_0 = 1.5$ ft $= 0.4572$ m and shallow-water $h_s = 0.5$ ft $= 0.1524$ m wavelengths. The wall condition or the so-called mirror condition $\zeta_y = 0$ is employed for side boundaries. At the end of the domain as the outgoing boundary condition a simplified form of Eq.(1) is used to radiate the waves away from within the domain.

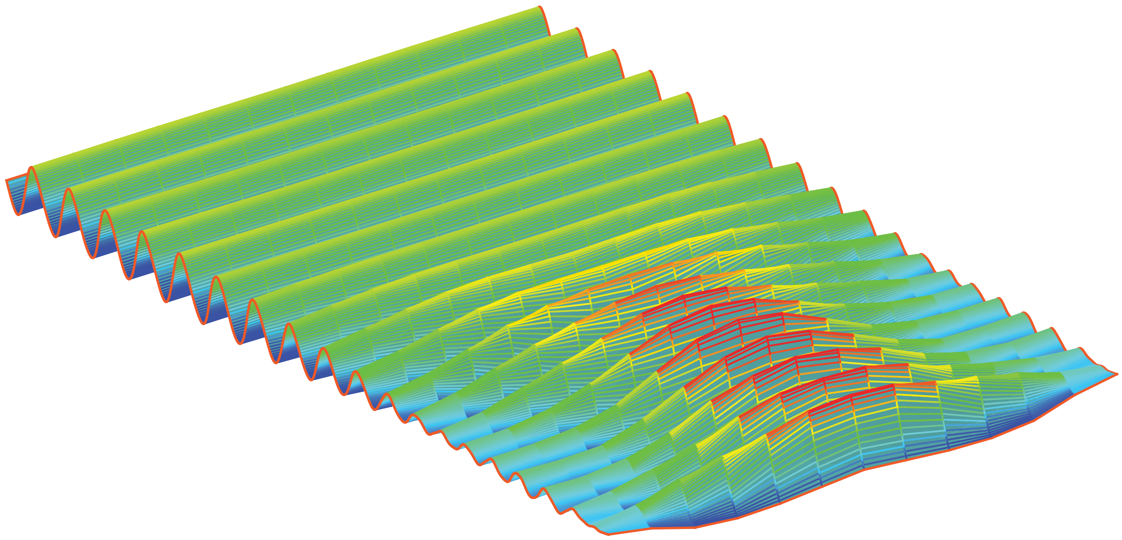

FIGURE 2. A perspective view of the fully-developed numerical wave tank for experiment reported in Whalin (1971).

CONCLUDING REMARKS

A generalized Kadomtsev-Petviashvili type equation for uneven water depths is presented. The new equation has improved linear dispersion and linear shoaling characteristics that extend its applicable range to virtually deep water waves. In particular, linear dispersion and shoaling characteristics of the equation are quite satisfactory, allowing accurate estimation of wave heights over varying depths ranging from relatively deep to very shallow depths. The wave model may be used for accurate estimation of wave conditions in nearshore regions.

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