

# Development and Testing of a Consistent Body Mass Index

Introducing a New Body Mass Index

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Biruni University

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by

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# Preface

*This report was compiled out of a research work that spanned more than one year between August 2024 and October 2025. The work provides a satisfactory answer to the longstanding problem of a body mass index which is truly scalable, in accord with geometrical fullness/slenderness concept, and equally usable for all ages and sexes.*

*Serdar Beji, Nezihe Kızılkaya Beji, Ümmü Mutlu  
Istanbul, March 2026*

# Summary

**Background:** The customarily used Body Mass Index  $BMI = m/h^2$  of Quetelet or its variant Tri-ponderal Mass Index  $TMI = m/h^3$  of Rohrer or other measures serving to the purpose of classifying the fitness level of a person have repeatedly been demonstrated to be unsatisfactory in various aspects. A new index inclusive of all ages, body sizes, and sexes is therefore needed.

**Methodology:** A Consistent Body Mass Index  $CBMI = \sqrt{m/h^3}$  is developed from first principles to classify the fitness level of a person according to the mass and height. The new index is essentially non-dimensional, well correlated with waist-to-height ratio,  $w/h$ , and equally valid for adults, children, and even infants of both sexes. Correlation levels of the CBMI and TMI (non-dimensional BMI) against waist-to-height ratio  $w/h$  are tested for subgroups and entire participants ranging from 1-day-old to 75-year-old males and females. Classification levels for  $w/h$  and CBMI indexes are determined according to the statistical characteristics of total samples.

**Findings:** Correlation coefficient computed for waist-to-height ratio  $w/h$  versus CBMI is highly satisfactory  $r = 0.84$  for the entire data set in comparison with the uncorrelated result of TMI, which is the non-dimensional form of the standard BMI. Two new classification charts, based on  $w/h$  and CBMI indexes, are developed for tentative classifications of the fitness levels.

**Interpretation:** Waist-to-height ratio  $w/h$  which correlates well with the newly introduced CBMI is interpreted as a reliable measure of the fitness of a person. Waist-to-height ratio  $w/h$  or CBMI can be used with equal confidence.

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# Nomenclature

## Abbreviations

Abbreviation	Definition
BAI	Body Adiposity Index
BIA	Bioelectrical Impedance Analysis
BMI	Body Mass Index
CBMI	Consistent Body Mass Index
CI	Corpulence Index
DXA (DEXA)	Dual-Energy X-ray Absorptiometry
FORTTRAN	Formula Translating System
PI	Ponderal Index
SBSI	Surface-based Body Shape Index
TMI	Tri-ponderal Mass Index (also CI & PI)
WC	Waist Circumference

## Symbols

Symbol	Definition	Unit
$h$	Height	[m]
$m$	Mass	[kg]
$N$	Number of data points	[-]
$p$	Percentage of adipose	[%]
$r$	Correlation coefficient and Radius	[-], [m]
$S_r$	Residual error of computations	[-]
$S_t$	Residual error of measurements	[-]
$\mathcal{V}$	Volume	[m <sup>3</sup> ]
$w$	Waist circumference	[m]
$w/h$	Waist circumference to height ratio	[-]
$\alpha$	Dimensional form factor of CBMI	[(m <sup>3</sup> /kg) <sup>1/2</sup> ]
$\alpha^*$	Non-dimensional form factor of CBMI	[-]
$\beta$	Dimensional form factor of TMI	[(m <sup>3</sup> /kg)]
$\beta^*$	Non-dimensional form factor of TMI	[-]
$\epsilon_T^2$	Total squared error	[-]
$\pi$	A constant	3.1415926...
$\rho$	Density	[kg/m <sup>3</sup> ]

# 1

## Introduction

In his book *L'homme moyen* (The average man) Lambert Adolphe Jacques Quetelet (1796-1874), astronomer, mathematician, and sociologist, introduced a measure, which was later termed as the Body Mass Index [1], BMI, in 1972. Although Quetelet did not specifically aim at developing an index, his definition, body mass divided by height squared  $m/h^2$ , gained recognition and became popular due probably to an obvious need for such a classification parameter. Despite the worldwide usage of Quetelet's body mass index, it has the serious drawback of not being directly applicable to children and youth, let alone infants. Likewise, the index is quite unreliable for relatively short or tall people. This particular defect originates from the dimensional form of the BMI ( $\text{kg}/\text{m}^2$ ), which renders it unscalable. Drawbacks associated with the BMI are discussed in several studies [2, 3, 4].

Such and similar problems have led to various different indexes such as the one proposed by Fritz Rohrer (1888–1926), the Corpulence Index (CI), also termed Ponderal Index (PI) or Tri-ponderal Mass Index (TMI). The TMI is defined as mass per cubic height  $m/h^3$  hence, when divided by a supposedly constant body density  $\rho$ , produces a non-dimensional parameter  $m/\rho h^3$ . While the non-dimensional form of the TMI definitely improves its applicable range as reported in various studies [5, 6], it does not suffice to make the TMI a universally acceptable index as it cannot be related to a geometrically meaningful measure of fitness; specifically, waist circumference to height ratio  $w/h$ .

Another choice for the BMI is the so-called new BMI which uses a flexible exponent  $E$  for the height  $m/h^E$  to overcome one of the major problems of the BMI index related to persons relatively short or tall. Accordingly, Trefethen [7] suggested to use the exponent of  $E = 2.5$  and a scaling factor of 1.3 so that the new BMI reads  $1.3 \cdot m/h^{2.5}$ . In this formula the exponent is selected as an intermediate between the BMI and TMI and the scaling factor 1.3 aligns the new BMI with the standard BMI in terms of numerical values. The new BMI is then a compromise between the dimensionally incorrect BMI and dimensionally correct TMI to remedy the problems associated with people of shorter or taller heights. To clarify the underlying motivation Trefethen [7] quotes from Quetelet's book of 1842 "A Treatise on Man and the Development of his Faculties":

*"If man increased equally in all dimensions, his weight at different ages would be as the cube of his height. Now, this is not what we really observe. The increase of weight is slower, except during the first year after birth; then the proportion we have just pointed out is pretty regularly observed. But after this period, and until near the age of puberty, weight increases nearly as the square of the height. The development of weight again becomes very rapid at puberty, and almost stops after the twenty-fifth year. In general, we do not err much when we assume that during development the squares of the weight at different ages are as the fifth powers of the height; which naturally leads to this conclusion, in supporting the specific gravity constant, that the transverse growth of man is less than the vertical."*

The main argument above clearly centers around the increase of weight in relation to height *in time*. Therefore, the formulation bases itself on the comparative temporal growth rates of weight and height. Such an approach, although sophisticated, is unfortunately misleading since the principal aim should simply be the determination of a measure of fullness or slenderness in terms of weight and height *at any given time* but not as a function of time. Further, a body mass index must also be commensurate with a non-dimensional geometric parameter such as waist-to-height ratio.

Returning to the connection between the Quetelet's remarks and the new BMI we note the sentence "the squares of the weight at different ages are as the fifth powers of the height," which obviously formulates to  $m^2/h^5$ , and by taking the square root, can be expressed as  $m/h^{2.5}$ . Nevertheless, as indicated by Trefethen, Quetelet adhered to the second power of height  $h^2$  and defined his measure as it is currently used  $m/h^2$ . The new BMI on the other hand, while apparently removing some of the problems associated with Quetelet's original index, still inherits the fundamental problem of dimensionality and because of this, cannot be related to a geometrically meaningful non-dimensional parameter.

Body Adiposity Index (BAI), Surface-based Body Shape Index (SBSI), and simply waist circumference (WC) may be counted among several other indexes. All these however have in common a dimensionality which renders scalability unattainable. For a universal applicability or scalability and mathematical consistency any such classification measure must be non-dimensional. In this work, starting from first principles, we derive a body mass index, which essentially differs from the Triponderal Mass Index (TMI) by a square root. This apparently minor difference however gives rise to very substantial improvements as demonstrated by correlation calculations based on waist circumference to height ratios.

In 2011 WHO Expert Consultation published a report entitled Waist Circumference and Waist-Hip Ratio [8] compiled by WHO experts in December 2008. The report emphasizes the importance of waist circumference and waist-hip ratio measures in identifying metabolic diseases and risk factors concerning cardiovascular diseases, hypertension, diabetes, various cancer types (colorectum, breast, kidney, etc.), and finally mortality. All these risks are associated with and classified according to the BMI and waist circumference. Unfortunately, the classifications based on the standard dimensional BMI ( $\text{kg}/\text{m}^2$ ) and dimensional waist circumference (m) cannot be relied on for wide range of samples. Clearly, while a waist size of 0.9 m for a man of 1.80 m height may be classified fit, the same waist size would not be considered so for a man

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of 1.60 m height. A non-dimensional parameter, waist-to-height ratio, would be much more reliable in such classifications as dimensional analysis teaches us [9]. Indeed, relatively recent studies consider non-dimensional measures such as waist-to-hip ratio [10, 11] and more appropriately waist-to-height ratio [12-14] rather than body mass index BMI for predicting various health risks and years of life lost due to obesity. Thus, adhering to first principles in connection with dimensional analysis and relating the body mass index to waist-to-height ratio, a consistent body mass index, which correlates highly with waist-to-height ratio for widely scattered body sizes and ages, is developed here.

Chapter 2 presents the derivation of consistent body mass index by simplifying the human body as a circular cylinder with a height equal to the height of the individual and radius equal to an overall-average waist radius. CBMI is then formulated as a non-dimensional quantity proportional to the waist-to-height ratio, which is a geometric measure of slenderness or fullness of a body. The standard BMI on the other hand is re-expressed as a parameter commensurate with the waist-to-height ratio and thereby shown that, when viewed from correct dimensional perspective, BMI is indeed equivalent to TMI. Measurements conducted specifically for the present study and relevant statistics are presented for separate subgroups and the entire data set. Chapter 2 ends with a brief description of the method of least squares as applied to the present problem.

Chapter 3 gives the correlation calculations of the body mass indexes versus waist-to-height ratios for CBMI and the non-dimensionalized BMI or equivalently TMI, separately. Superiority of CBMI over BMI/TMI is visually observed from the graphs depicting the correlations in various groups of samples. Quantitatively, the same conclusion is confirmed by the residual errors and correlation coefficients computed for CBMI and BMI/TMI. Based on the waist-to-height-ratio and CBMI index separate fitness criteria and corresponding charts are developed according to the statistical calculations done for the entire measurement data.

The final chapter is devoted to a discussion of results and possible future refinements of the index developed.

# 2

## Methodology

### Formulation of Index from First Principles

As remarked in Chapter 1, Quetelet's 1842 book refers to different stages of body development with different growth rates and argues the merits of different powers of height. This viewpoint is nevertheless irrelevant because the sole purpose of a fitness index is to provide a quantitative measure for the bodily state of a person at any given time without attempting to model growth rates. Furthermore, the index must be scalable, or equivalently non-dimensional, and relatable to another non-dimensional quantity geometrically describing the fullness or slenderness of an object; namely, the width to the length ratio. To this end, we begin with the elementary definition that mass is equal to density times volume:

$$m = \rho \mathcal{V}, \quad (2.1)$$

where  $m$  is mass in kg,  $\rho$  density in  $\text{kg}/\text{m}^3$ , and  $\mathcal{V}$  volume in  $\text{m}^3$  according to the SI units. Assuming that the human body may be approximated as a circular cylinder whose height is the same as a person's height  $h$  and whose radius  $r$  is equal to the *overall average* waist radius of a person as sketched in Figure 1, the cylindrical volume  $\mathcal{V}$  is

$$\mathcal{V} = \pi r^2 h = \pi (w/2\pi)^2 h = \frac{1}{4\pi} w^2 h, \quad (2.2)$$

where  $w = 2\pi r$  is taken as the average waist circumference corresponding to the radius  $r$  defined above. Substituting  $\mathcal{V}$  into the definition of mass gives

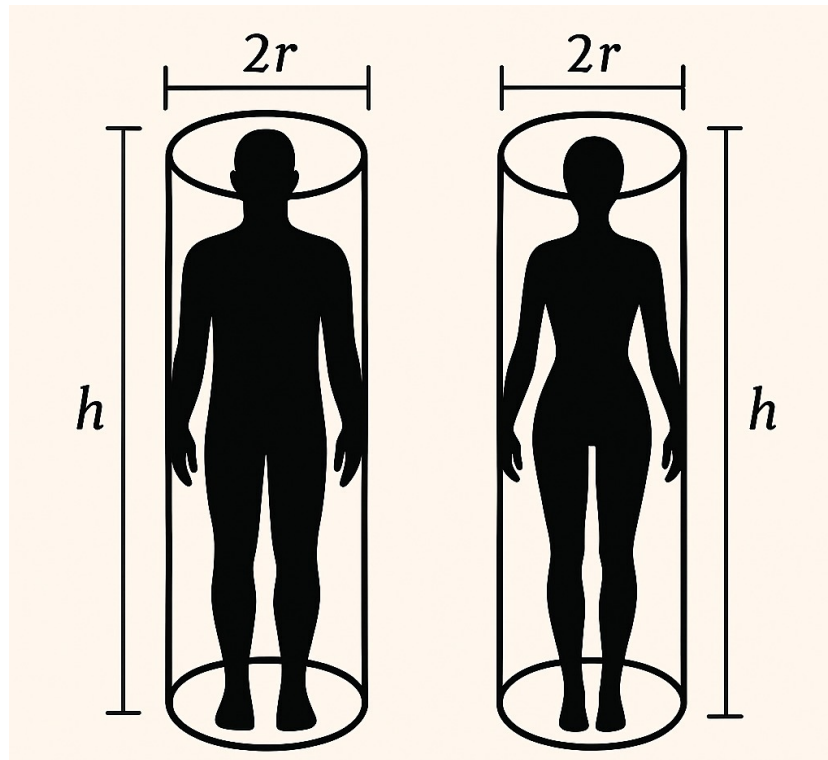
$$m = \rho \mathcal{V} = \frac{\rho}{4\pi} w^2 h, \quad (2.3)$$

and solving for  $w$  yields

$$w = 2\sqrt{\frac{\pi m}{\rho h}}. \quad (2.4)$$

In order to formulate a dimensionless measure of fitness both sides of the above equation are divided by  $h$  so that

$$\frac{w}{h} = 2\sqrt{\frac{\pi m}{\rho h^3}}. \quad (2.5)$$



**Figure 2.1:** Representation of human body as a circular cylinder with radius  $r$  and height  $h$ . Geometric measure of slenderness or fullness of an object: Waist-to-height ratio  $w/h = \pi(2r/h)$  (width/height), where waist circumference  $w = 2\pi r$ .

Finally, to account for the differences in forms of human bodies from that of a circular cylinder we define a non-dimensional coefficient  $\alpha^*$ , which may be termed as the *form factor*, so that

$$\frac{w}{h} = 2\alpha^* \sqrt{\frac{\pi m}{\rho h^3}} = \alpha \sqrt{\frac{m}{h^3}}, \quad (2.6)$$

where  $\alpha = 2\alpha^* \sqrt{\pi/\rho}$  is a dimensional constant introduced to factor out the relatively smaller effect of changes in body density from person to person. Thus, the Consistent Body Mass Index CBMI is defined as

$$\text{CBMI} = \sqrt{\frac{m}{h^3}} \quad (2.7)$$

which is directly related to the ratio of waist circumference to the height,  $w/h = \alpha \cdot \text{CBMI}$ , probably the most important parameter measuring the fitness level of a person as it provides a relative measure applicable to all body sizes, ages, and sexes as demonstrated in Chapter 3. Waist-to-height ratio  $w/h$  is then related to the consistent body mass index  $\text{CBMI} = \sqrt{m/h^3}$  as

$$\frac{w}{h} = \alpha \sqrt{\frac{m}{h^3}} \quad (2.8)$$

The CBMI defined in equation (2.7) does not appear non-dimensional; but this is only apparent because if the body density  $\rho$  were kept,  $\sqrt{m/\rho h^3}$ , it would be explicitly non-dimensional.

Human body density shows variations depending on sex and percentages of fatty, muscular, and bone tissues; therefore, the dimensional constant  $\alpha = 2\alpha^* \sqrt{\pi/\rho}$  would in principle differ according to sex, tissue composition, and to some extent age.

If  $p$  indicates the percentage of adipose (fat) tissue the average density of body can simply be calculated by a weighted averaging formulation  $\rho = 1/[p/\rho_1 + (1-p)/\rho_2]$  where  $\rho_1 = 901 \text{ kg/m}^3$  and  $\rho_2 = 1100 \text{ kg/m}^3$  are the fat and dry mass densities, respectively. Using the average percentages of fat for men  $p = 0.155$  and for women  $p = 0.225$  in the above formula gives the average densities  $\rho_M = 1063.6 \text{ kg/m}^3$  for men and  $\rho_W = 1047.9 \text{ kg/m}^3$  for women, which indicates a difference of approximately 1.5%. The average human body density is then  $\rho = 1056 \text{ kg/m}^3$ ; however, for simplicity, taking the human body density approximately  $\rho = 1000 \text{ kg/m}^3$  gives  $\alpha = 0.11\alpha^*$  where the multiplier 0.11 is a dimensional quantity and makes both sides of equation (2.8) non-dimensional. Further, using the average value  $\alpha = 0.129$  obtained in Chapter 3,  $\alpha^* = \alpha/0.11$  becomes  $\alpha^* = 1.17$  or  $\alpha^* \sim O(1)$ , and indicates that the constructed model is good.

A more accurate index may be derived by including the density variations as a function of waist-to-height ratio  $\rho = \rho(w/h)$  or equivalently the consistent body mass index  $\rho = \rho(\text{CBMI})$ . To keep the present formulation as simple as possible, presently no such attempt is made; nevertheless, trial computations for male and female adults resulted in even better correlation coefficients between  $w/h$  and CBMI values, hence provided support to pursue this idea. In the future, a refinement in this direction is planned via semi-empirical formulations of  $\rho = \rho(\text{CBMI})$  for men, women, and children, separately.

## Standard BMI Expressed as TMI and Related to Waist-to-Height Ratio

The newly defined body mass index, being in essence non-dimensional, has been linked to waist-to-height ratio, a meaningful measure of the body fitness as well as health risks. The standard BMI =  $m/h^2$  on the other hand has the dimension of mass over length squared ( $\text{kg/m}^2$ ) and does not look promising for establishing a connection with waist-to-height ratio. However, dividing the standard BMI by the body density  $\rho$ , a presumably constant quantity, gives  $m/\rho h^2$  which has the dimension of length (m). Such a length may naturally be assumed to be *proportional* to the waist circumference, as implied in the WHO report [8] which related these quantities. Then,

$$w \propto \frac{m}{\rho h^2} \quad (2.9)$$

where both sides of the above expression are *dimensionally in accord* and may be written as an equality by introducing a non-dimensional proportionality constant, or *form factor*, say  $\beta^*$ :

$$w = \beta^* \frac{m}{\rho h^2} \quad (2.10)$$

Due to the advantages of using non-dimensional quantities as well as performing consistent comparisons of BMI with CBMI we now rewrite the above equality as  $w/h$

by simply dividing both sides by  $h$ :

$$\frac{w}{h} = \beta \frac{m}{h^3} \quad (2.11)$$

where  $\beta = \beta^*/\rho$  is a dimensional constant absorbing the body density  $\rho$  as in the definition of  $\alpha$ . The right-hand side of equation (2.11) is nothing other than the tri-ponderal mass index TMI. Now that BMI and TMI are clearly related we are ready to compare equation (2.8) with (2.11) in terms of their performances in predicting measured  $w/h$  values from the computed CBMI =  $\sqrt{m/h^3}$  and TMI =  $m/h^3$  values. CBMI differs from TMI by the square root alone; but this fine difference results in remarkable improvements in correlation coefficients computed for  $w/h$  versus CBMI values in comparison with those obtained for  $w/h$  versus TMI.

## Measurements

The required ethical permission to conduct the data collection part of the study was approved and issued by Biruni University Ethics Commission (Ruling no. 2024-BIAEK/09-59, 21 April 2025). Hospital staff working at Biruni University Hospital, healthy volunteers, patients' relatives and/or children accompanying the patients and infants in the neonatal unit were included in the study. Participants were not subjected to any prioritized selection criteria other than being healthy without any chronic or other diseases. The process was completely randomized based on available and consenting individuals. All the participants were measured in the same way after consent was obtained. Weight<sup>1</sup> was measured with a scale placed on an equal surface while the participants were barefoot and wearing light clothes, height was measured barefoot in upright position, infants in supine position, and waist circumference was measured with measuring tape, the midpoint between the lower border of the rib cage and the iliac crest at the end of normal expiration. Measurements were performed via the available standard equipment of Biruni University with an accuracy of 0.1 kg for mass and 1 cm for length.

## Statistical Analyses

Basic statistical analyses were carried out on the Excel data files established for the subgroups and entire participants. 400 participants were measured in four subgroups: teenage and adult females, teenage and adult males, female children and male children, each subgroup containing 100 participants. Teenagers and adults were classified together as adults for ages 10 years and older; consequently, less than 10-year-olds were categorized as children. Data for four groups; female adults, male adults, female children, and male children are separately given in Appendix A.

Female adults range between minimum 10 years and maximum 70 years. The average age of the group is 30 years with standard deviation  $\pm 12$  years, the average weight  $63 \pm 15$  kg, the height  $163 \pm 10$  cm, the waist circumference  $80 \pm 14$  cm, and the  $w/h$

<sup>1</sup>Weight normally indicates a quantity with dimension of force or newton in SI units. However, in the relevant literature "weight" is customarily referred to as "mass" in kg in SI units. Accordingly, we use "weight" and "mass" interchangeably with the SI unit of kg.

**Table 2.1:** Basic statistical characteristics of measured data as averages $\pm$ standard deviations.

Group	Age range	Age (year)	Mass (kg)	Height(cm)	Waist (cm)	$w/h$
Female adults	10 – 70	$30 \pm 12$	$63 \pm 15$	$163 \pm 10$	$80 \pm 14$	$0.49 \pm 0.08$
Male adults	10 – 75	$33 \pm 18$	$69 \pm 21$	$168 \pm 15$	$84 \pm 15$	$0.50 \pm 0.07$
Female children	0 – 9	$5 \pm 3$	$18 \pm 8$	$103 \pm 25$	$51 \pm 8$	$0.51 \pm 0.07$
Male children	0 – 9	$6 \pm 3$	$21 \pm 8$	$110 \pm 21$	$54 \pm 8$	$0.50 \pm 0.06$
All	0 – 75	$18 \pm 17$	$43 \pm 27$	$136 \pm 35$	$67 \pm 18$	$0.50 \pm 0.07$

ratio  $0.49 \pm 0.081$ . Male adults are in the range of 10 to 75-year-olds with the average age  $33 \pm 18$  years, the mass  $69 \pm 21$  kg, the height  $168 \pm 15$  cm, the waist circumference  $84 \pm 15$  cm, and the  $w/h$  ratio  $0.50 \pm 0.065$ . Female children range from 1-day-olds (newborns) to 9-year-olds with the average age  $5 \pm 3$  years, the mass  $18 \pm 8$  kg, the height  $103 \pm 25$  cm, the waist  $51 \pm 8$  cm, and the  $w/h$  ratio  $0.51 \pm 0.068$ . Male children range from 1-day-olds (newborns) to 9-year-olds with the average age  $6 \pm 3$  years, the mass  $21 \pm 8$  kg, the height  $110 \pm 21$  cm, the waist  $54 \pm 8$  cm, and the  $w/h$  ratio  $0.50 \pm 0.063$ . Altogether for the entire 400 participants the ages run between 1-day-olds and 75-year-olds with the average  $18 \pm 17$  years, the mass  $43 \pm 27$  kg, the height  $136 \pm 35$  cm, the waist  $67 \pm 18$  cm, and the  $w/h$  ratio  $0.50 \pm 0.068$ . Table 2.1 gives all the relevant statistical figures as averages $\pm$ standard deviations. Note that except for the non-dimensional  $w/h$  ratio, the standard deviations for all the items, but in particular for the ages, are very large, reflecting the widely scattered range of samples used for testing the applicability of CBMI.

Table 2.1 clearly confirms that the waist circumference to the height ratio  $w/h$  is the right choice for classifying the body characteristics as its mean value is virtually constant regardless of age and sex and despite the wide range of variations observed in all the other dimensional figures. Furthermore, this purely geometric ratio is connected to the essentially non-dimensional physical scale CBMI as given in equation (2.8) and CBMI can too be used as a separate classification index by itself as demonstrated in Chapter 3.

## Application of Method of Least Squares

The main hypothesis of the present work assumes a direct proportionality between waist-to-height ratio  $w/h$  and the newly introduced body mass index  $CBMI = \sqrt{m/h^3}$ . Both these quantities are non-dimensional and that CBMI is termed consistent because it accords well with the geometric non-dimensional parameter  $w/h$ . In order to quantitatively determine the degree of correlation between  $w/h$  and CBMI as well as

between  $w/h$  and TMI we first recall equations (2.8) and (2.11):

$$\frac{w}{h} = \alpha \sqrt{\frac{m}{h^3}}, \quad \frac{w}{h} = \beta \frac{m}{h^3}. \quad (2.12)$$

The method of least-squares is employed to determine form factors  $\alpha$  and  $\beta$  separately for the data sets collected. Briefly, the method is applied by minimizing the sum of errors squared for each data point with respect to the proportionality constant or form factor. Total squared error  $\epsilon_T^2$  for the measured  $w/h$  values against the predicted values of CBMI is

$$\epsilon_T^2 = \sum_{i=1}^N \epsilon_i^2 = \sum_{i=1}^N \left( \frac{w_i}{h_i} - \alpha \sqrt{\frac{m_i}{h_i^3}} \right)^2, \quad (2.13)$$

where  $N$  is the number of data points. The total error is minimized by taking the partial differentiation of  $\epsilon_T^2$  with respect to  $\alpha$  and equating the result to zero so that  $\alpha$  is calculated as

$$\frac{\partial \epsilon_T^2}{\partial \alpha} = -2 \sum_{i=1}^N \sqrt{\frac{m_i}{h_i^3}} \left( \frac{w_i}{h_i} - \alpha \sqrt{\frac{m_i}{h_i^3}} \right) = 0 \quad \Rightarrow \quad \alpha = \frac{\sum_{i=1}^N (w_i/h_i) \sqrt{m_i/h_i^3}}{\sum_{i=1}^N m_i/h_i^3}. \quad (2.14)$$

Once  $\alpha$  is computed the predicted values are obtained from  $\alpha \sqrt{m_i/h_i^3}$  and since the measured values of  $w_i/h_i$  are known the total residual error  $S_r$  is computed as  $S_r = \sum_{i=1}^N \left( w_i/h_i - \alpha \sqrt{m_i/h_i^3} \right)^2$ . The residual error of the measured  $w_i/h_i$  values,  $S_t$ , is calculated from  $S_t = \sum_{i=1}^N [w_i/h_i - (w_i/h_i)_{\text{mean}}]^2$ . Accordingly, the extent of the improvement provided by the model equation CBMI is quantified by the correlation coefficient  $r = \sqrt{(1 - S_r/S_t)}$ . Note that when the residual error  $S_r = 0$ , the correlation coefficient  $r = 1$  hence the model represents the data perfectly. On the other hand, if  $S_r \geq S_t$  the model completely fails to represent the data. For TMI exactly the same approach is applied to obtain  $\beta$ ; only  $m_i/h_i^3$  is used in place of  $\sqrt{m_i/h_i^3}$ . The results presented in Chapter 3 are obtained according to these formulations by the use of a FORTRAN code developed specifically for this application. The code is given in the Appendix B.

# 3

## Results

The present work establishes a direct connection between two non-dimensional quantities; namely, waist-to-height ratio  $w/h$  and consistent body mass index  $\text{CBMI} = \sqrt{m/h^3}$ . Therefore, the degree of correlation between these quantities must be checked. In the same vein, the correlation of  $w/h$  ratio with  $\text{TMI} = m/h^3$  must be investigated for comparing the performances of  $\text{CBMI}$  and  $\text{TMI}$ . The comparisons clearly show that  $\text{CBMI}$  correlates with waist-to-height ratio much better than  $\text{TMI}$  and can be used as a reliable measure of fitness since waist-to-height ratio itself, being virtually constant on the average 0.5 for all ages and sexes, can be taken as the true measure of fitness level.

### Waist-to-Height versus $\text{CBMI}$ and $\text{TMI}$

In accord with the principal idea that a reliable body mass index must be well compatible with waist-to-height ratio we first compute  $\alpha$  and  $\beta$  values connecting  $\text{CBMI}$  and  $\text{TMI}$  to  $w/h$  by the method of least squares described in Chapter 2. Then, measured  $w/h$  values versus corresponding  $\text{CBMI}$  and  $\text{TMI}$  values as computed from equations given in (2.12) are plotted separately and relevant correlation coefficients are computed. In the calculations, adhering to the SI units, weights are taken in kilogram (kg), heights and waist circumferences in meter (m). Computations are carried out for the four subgroups of 100 participants each and for the entire 400 participants, separately.

**Table 3.1:** Correlation characteristics of  $w/h$  versus  $\text{CBMI}$ .

CBMI	Form factor $\alpha$	Residual Error (model) $S_r$	Residual Error (data) $S_t$	Correlation coefficient $r$
Female adults	0.128	0.159	0.540	0.840
Male adults	0.133	0.185	0.415	0.744
Female children	0.128	0.082	0.463	0.907
Male children	0.128	0.099	0.391	0.864
All	0.129	0.555	1.851	0.837

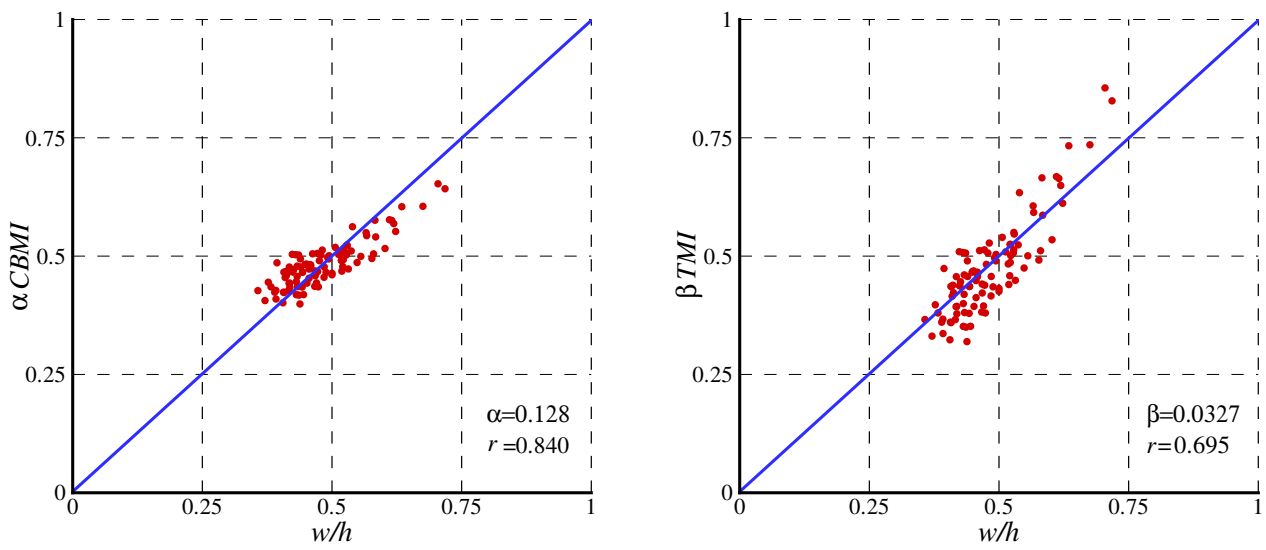
**Table 3.2:** Correlation characteristics of  $w/h$  versus TMI.

TMI	Form factor $\beta$	Residual Error (model) $S_r$	Residual Error (data) $S_t$	Correlation coefficient $r$
Female adults	0.0327	0.279	0.540	0.695
Male adults	0.0351	0.209	0.415	0.704
Female children	0.0304	0.747	0.463	N.A.
Male children	0.0310	0.833	0.391	N.A.
All	0.0321	2.373	1.851	N.A.

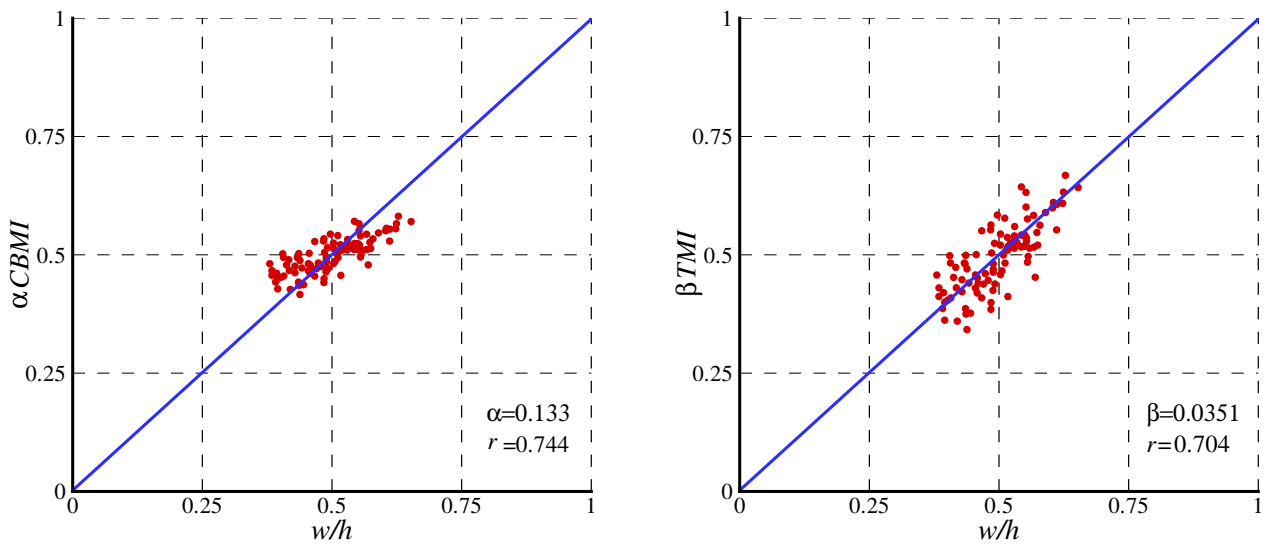
Statistical results for measured  $w/h$  values versus corresponding predicted values as computed from  $\alpha \cdot \text{CBMI}$  and  $\beta \cdot \text{TMI}$  are summarized in Table 3.1 and Table 3.2, respectively. As seen in Table 3.1, use of CBMI produces a form factor  $\alpha = 0.128$  and correlation coefficient  $r = 0.840$  for female adults,  $\alpha = 0.133$  and correlation coefficient  $r = 0.744$  for male adults,  $\alpha = 0.128$  and correlation coefficient  $r = 0.907$  for female children, and  $\alpha = 0.128$  and correlation coefficient  $r = 0.864$  for male children. For the entire data containing all the participants, female and male, adult and children  $\alpha = 0.129$  and correlation coefficient  $r = 0.837$ . Form factors for the three groups are the same and this point is clearly reflected to the overall form factor  $\alpha = 0.129$  which deviates 3.1% from  $\alpha = 0.133$  of male adults. The correlation coefficients range from the lowest  $r = 0.744$  for male adults to the highest  $r = 0.907$  for female children. All these values are good enough to confirm validity of the initial hypothesis of relating  $w/h$  and CBMI; nevertheless, compared to the rest, notable deviations in the form factor and correlation coefficient of male adults are obvious. Clarification of these points require analyses from different aspects and therefore it is deferred to the discussion.

Table 3.2 shows the results for TMI. First, note that the residual errors computed for the measured data,  $S_t$ , are the same for all the items in both Table 3.1 and Table 3.2 since the data used is the same. For female adults the form factor  $\beta = 0.0327$  and correlation coefficient  $r = 0.695$ ; for male adults  $\beta = 0.0351$  and  $r = 0.704$ ; for female children  $\beta = 0.0304$  and  $r$  cannot be computed (marked as Not Available, N.A.) because  $S_r > S_t$  hence  $r^2 < 0$ ; likewise, for male children  $\beta = 0.0310$  and again  $r^2 < 0$ , indicating that TMI does not correlate at all with  $w/h$  for children. Thus, from the mathematical point of view, TMI cannot be a valid body mass index for children. Finally, for the complete data we get  $\beta = 0.0321$  with no correlation  $r^2 < 1$ . Form factors of the subgroups differ as much as 13% between the male adults and female children; likewise, the overall form factor  $\beta = 0.0321$  deviates by 9% from  $\beta = 0.0351$  of male adults. The correlation coefficients could not be computed for children and for the entire data set. The highest correlation coefficient is  $r = 0.704$  for male adults. In view of these results, when compared with CBMI, TMI fairs quite poorly as a mass body index. This conclusion is also demonstrated visually by the graphs, which are presented next.

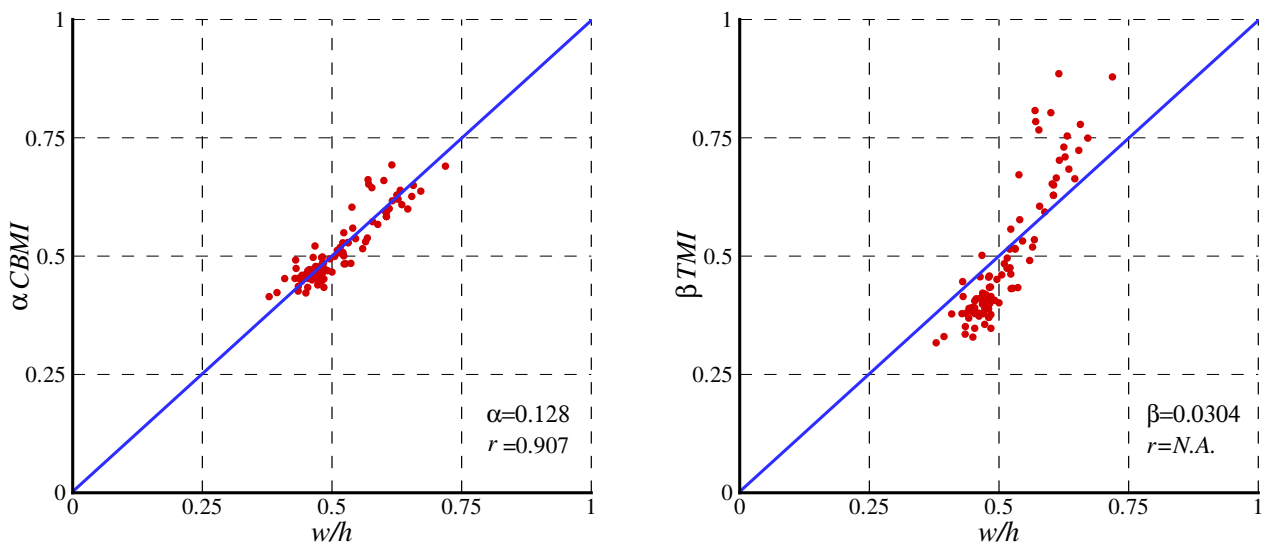
Figure 3.1 depicts respectively  $w/h$  versus  $\alpha \cdot \text{CBMI}$  and  $\beta \cdot \text{TMI}$  plots for female adults. From the graphs as well as from the correlation coefficients,  $r = 0.840$  and  $r = 0.695$ , it is clear that qualitatively and quantitatively CBMI correlates well with  $w/h$ . The corresponding plots for male adults are presented in Figure 3.2.



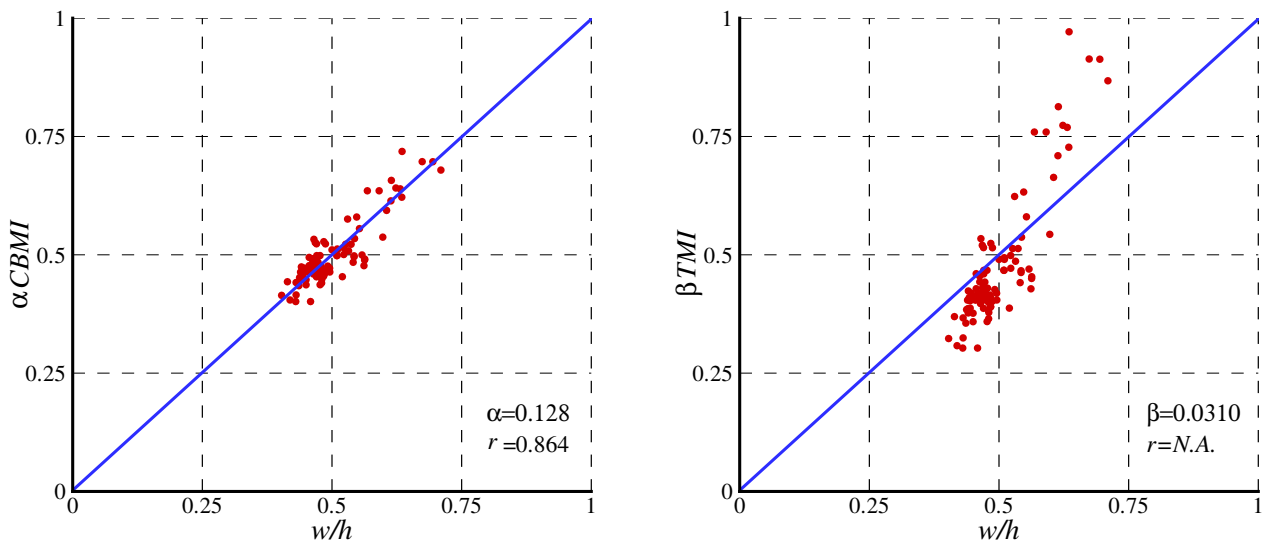
**Figure 3.1:** Waist-to-height ratio  $w/h$  versus  $\alpha \cdot CBMI$  (left,  $r = 0.840$ ) and  $\beta \cdot TMI$  (right,  $r = 0.695$ ) for female adults.



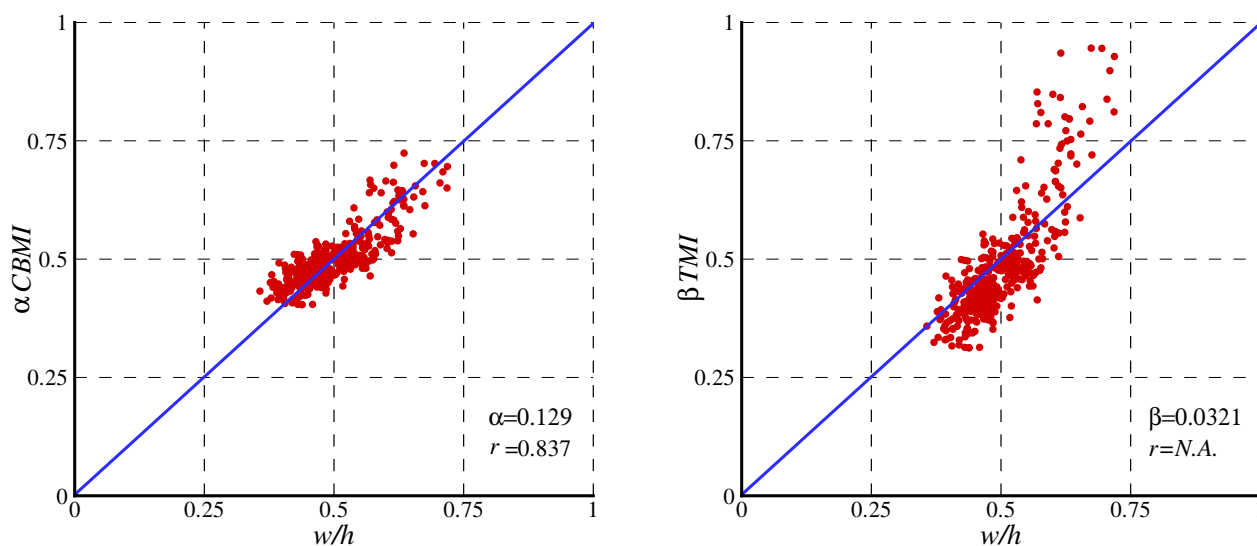
**Figure 3.2:** Waist-to-height ratio  $w/h$  versus  $\alpha \cdot CBMI$  (left,  $r = 0.744$ ) and  $\beta \cdot TMI$  (right,  $r = 0.704$ ) for male adults.



**Figure 3.3:** Waist-to-height ratio  $w/h$  versus  $\alpha \cdot CBMI$  (left,  $r = 0.907$ ) and  $\beta \cdot TMI$  (right,  $r = N.A.$ ) for female children.



**Figure 3.4:** Waist-to-height ratio  $w/h$  versus  $\alpha \cdot CBMI$  (left,  $r = 0.864$ ) and  $\beta \cdot TMI$  (right,  $r = N.A.$ ) for male children.



**Figure 3.5:** Waist-to-height ratio  $w/h$  versus  $\alpha \cdot CBMI$  (left,  $r = 0.837$ ) and  $\beta \cdot TMI$  (right,  $r = N.A.$ ) for entire data set.

Figure 3.3 shows the plots of  $w/h$  versus  $\alpha \cdot CBMI$  and  $\beta \cdot TMI$  for female children and Figure 3.4 for male children. Relatively higher waist-to-height ratios  $w/h > 0.6$  in these graphs are mainly the infant data rather than overweight children. In the same vein, big scatter in graphs of  $w/h$  versus  $\beta \cdot TMI$  especially for  $w/h > 0.6$  should be interpreted as the failure of TMI in quantitative modeling of children and infant data.

Finally, Figure 3.5 shows  $w/h - CBMI$  and  $w/h - TMI$  plots for the complete data set. Overall, good performance of CBMI as a body mass index quantitatively representing the corresponding geometric measure  $w/h$  of body slenderness/fullness is obvious.

## Waist-to-Height Ratio and CBMI

As demonstrated by computational results, a good correlation between waist-to-height ratios and corresponding CBMI values has been established. Table 3.3 gives the basic statistical results, average  $\pm$  standard deviation, for the measured waist-to-height ratios  $w/h$  and computed  $CBMI = \sqrt{m/h^3}$  values, which are regarded non-dimensional. A nearly constant proportionality factor between  $w/h$  and CBMI is obvious for all the groups.

Using the form factor  $\alpha = 0.129$  computed for the entire data set of 400 participants some sample estimates of  $w/h$  are now computed from  $\alpha \cdot CBMI$  and compared with the measured  $w/h$  values for each sub-group of 100 participants. Tables 3.4-3.7 give sample computations for wide range of age groups for female adults, male adults, female children, and male children, respectively.

**Table 3.3:** Basic statistical characteristics of consistent body mass index for different groups.

Group	Age range	Age (year)	Mass (kg)	Height(cm)	$w/h$	CBMI
Female adults	10 – 70	30 ± 12	63 ± 15	163 ± 10	0.49 ± 0.08	3.79 ± 0.39
Male adults	10 – 75	33 ± 18	69 ± 21	168 ± 15	0.50 ± 0.07	3.75 ± 0.27
Female children	0 – 9	5 ± 3	18 ± 8	103 ± 25	0.51 ± 0.07	3.97 ± 0.55
Male children	0 – 9	6 ± 3	21 ± 8	110 ± 21	0.50 ± 0.06	3.88 ± 0.52
All	0 – 75	18 ± 17	43 ± 27	136 ± 35	0.50 ± 0.07	3.85 ± 0.45

**Table 3.4:** Female adults: Sample comparisons of measured waist-to-height ratios and computed  $\alpha \cdot$  CBMI values.

Age (year)	Mass (kg)	Height(cm)	Waist (cm)	$w/h$	$\alpha \cdot$ CBMI	Error%
10	40	147	67	0.456	0.458	-0.4%
26	58	165	74	0.448	0.464	-3.4%
41	87	167	87	0.521	0.558	-7.0%
59	77	156	81	0.519	0.581	-11.9%
70	65	158	80	0.506	0.523	-3.4%

**Table 3.5:** Male adults: Sample comparisons of measured waist-to-height ratios and computed  $\alpha \cdot$  CBMI values.

Age (year)	Mass (kg)	Height(cm)	Waist (cm)	$w/h$	$\alpha \cdot$ CBMI	Error%
10	25	126	62	0.492	0.456	7.3%
25	61	168	77	0.458	0.462	-0.9%
42	90	182	95	0.522	0.498	4.5%
60	78	162	88	0.543	0.553	-3.3%
75	72	170	95	0.559	0.494	11.6%

**Table 3.6:** Female children: Sample comparisons of measured waist-to-height ratios and computed  $\alpha \cdot$  CBMI values.

Age (year)	Mass (kg)	Height(cm)	Waist (cm)	$w/h$	$\alpha \cdot$ CBMI	Error%
0 (1 day)	3.1	51	32	0.627	0.628	-0.2%
2	12	85	50	0.588	0.575	2.3%
4	18	108	52	0.481	0.491	-2.1%
7	22	118	58	0.492	0.476	3.2%
9	31	134	60	0.448	0.467	-4.2%

**Table 3.7:** Male children: Sample comparisons of measured waist-to-height ratios and computed  $\alpha \cdot \text{CBMI}$  values.

Age (year)	Mass (kg)	Height(cm)	Waist (cm)	$w/h$	$\alpha \cdot \text{CBMI}$	Error%
0 (1 day)	3.3	52	33	0.635	0.630	0.8%
2	14	96	49	0.510	0.517	-1.3%
4	16	102	48	0.471	0.505	-7.3%
7	22	118	57	0.483	0.476	1.5%
9	30	134	59	0.440	0.459	-4.3%

# 4

## New Criteria Based on Waist-to-Height Ratio and CBMI

The waist-to-height ratio, a geometric indicator of slenderness or fullness of a body, is shown in Table 2.1 to be a reliable non-dimensional parameter for classifying the body types regardless of ages and sexes. Accuracy of this parameter in assessing the risks related to years of life lost due to obesity and obesity-related cardiovascular diseases [12-14] provides additional support to it as a good indicator. Likewise, the WHO report [8] may be viewed as an attempt in the same direction, except the report uses the dimensional waist circumference, an unscalable measure, that varies widely from person to person and cannot be quite related to BMI, which itself is another dimensional measure. The present work satisfactorily relates the waist-to-height ratio to the CBMI and concludes that  $w/h$  or equivalently CBMI may be used as fitness level indicators and risk assessors. Accordingly, first a criterion and relevant classification based on the waist-to-height ratio  $w/h$  is proposed and subsequently another criterion using CBMI is developed with parallel classification levels to  $w/h$  ratio.

### **Criterion Based on Waist-to-Height Ratio**

Measurements performed in the present work are rather limited and basically serve for the purpose of comparing performances of different body mass indexes, CBMI and TMI, against  $w/h$ . Therefore, the classification intervals suggested here is tentative and subject to change, depending on the contents of available data.

For the entire data set Table 2.1 gives an average waist-to-height ratio  $w/h = 0.50$  and standard deviation 0.068. This average is obtained for 400 female and male subjects aged between 0 – 75 years, residing in the city of Istanbul; therefore, these samples should not be taken as representative of any other larger groups or in other regions of the world. More importantly, the average ratio  $w/h = 0.50$ , may not be taken as indicative of healthy or desired fitness level as it is well-known that across the globe in the past 50 years or so there is an uptrend in average body weight due to increased availability of food supplies and varieties combined with unhealthy eating habits [15]. Despite all these reservations, classification levels for waist-to-height ratio are set

**Table 4.1:** Fitness levels according to waist-to-height ratios.

Classification	$100 \cdot w/h$
Underweight	< 42
Normal	42 – 50
Overweight	50 – 58
Obese	58 – 66
Morbid obese	> 66

according to relevant statistics computed from the measurements of the present study. In the future, data comprising considerably larger groups with adipose measurements included may be used to accurately classify the fitness levels.

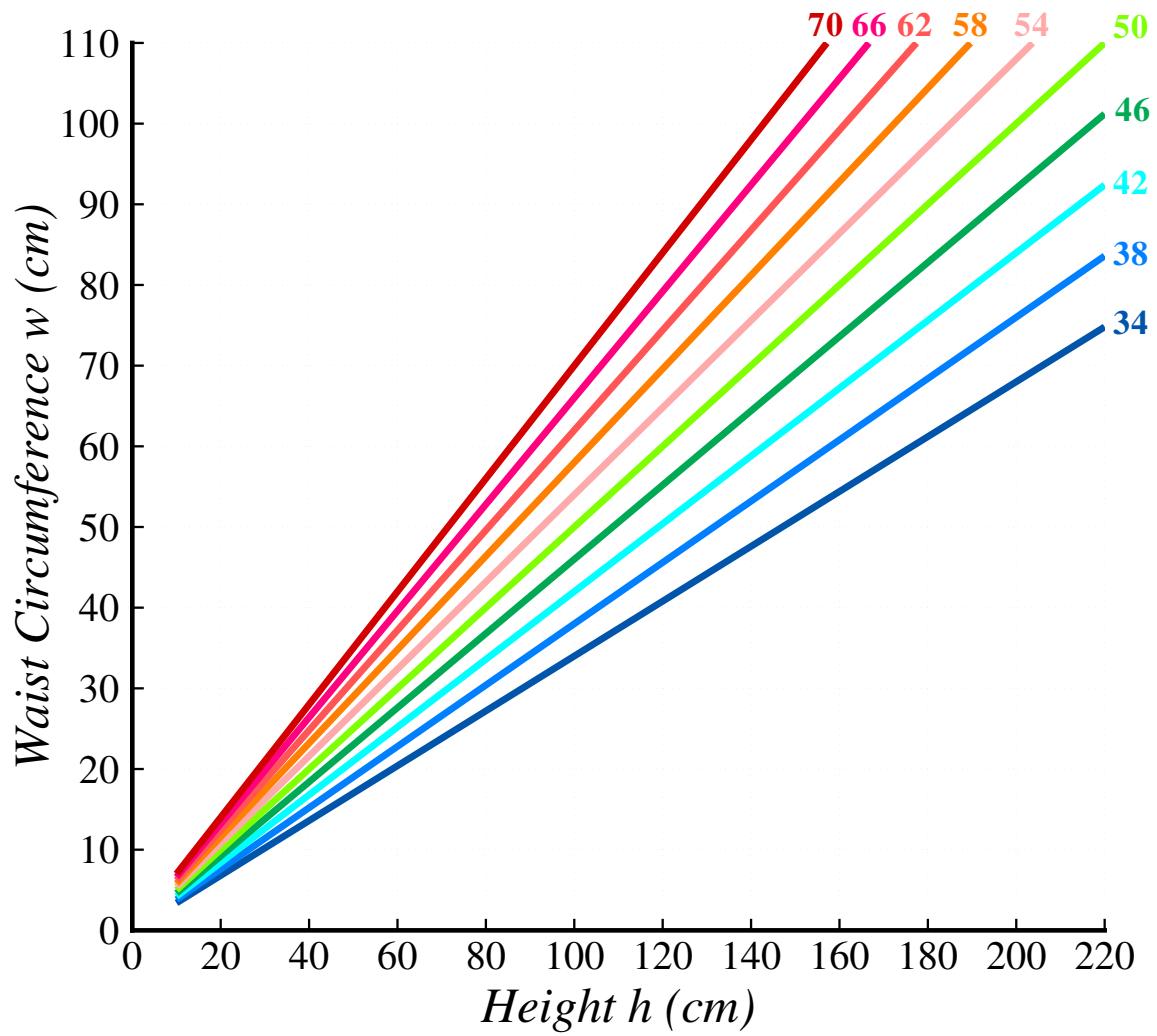
First, it is assumed that the average waist-to-height ratio found in this work  $w/h = 0.50$  is the upper limit of normal as suggested by the reported clinical results [12]. Typically, the lower and upper limits of the normal range could be defined by subtracting  $0.50 - 0.068 = 0.432$  and adding  $0.50 + 0.068 = 0.568$  the standard deviation 0.068 to the mean 0.50; however, a conservative viewpoint is adopted here and the ranges are determined by setting a lower range of deviation  $\pm 0.04$  with  $w/h = 0.46$  taken as the *mean normal* or *ideal normal* so that  $w/h = 0.50$  becomes the upper limit of normal as aimed. Finally, for ease of use in terms of integers, the range is re-scaled by using  $100 \cdot w/h$  instead of  $w/h$ . In calculations, taking the waist circumference in cm and height in m produces the desired result without the necessity of multiplying by 100.

Table 4.1 shows the classifications valid for any age and sex except for infants less than 1-year-old. A different scale, possibly one level shifted up (e.g., Normal range: 50 – 60) must be used for infants as their average  $w/h \approx 0.60$  regardless of sex. Once again, all these levels are tentatively proposed and may be redefined according to extended measurements of different groups and more importantly judgment of involved professionals. Figure 4.1, visual representation of Table 4.1, depicts the classification of fitness levels for definite waist-to-height ratios.

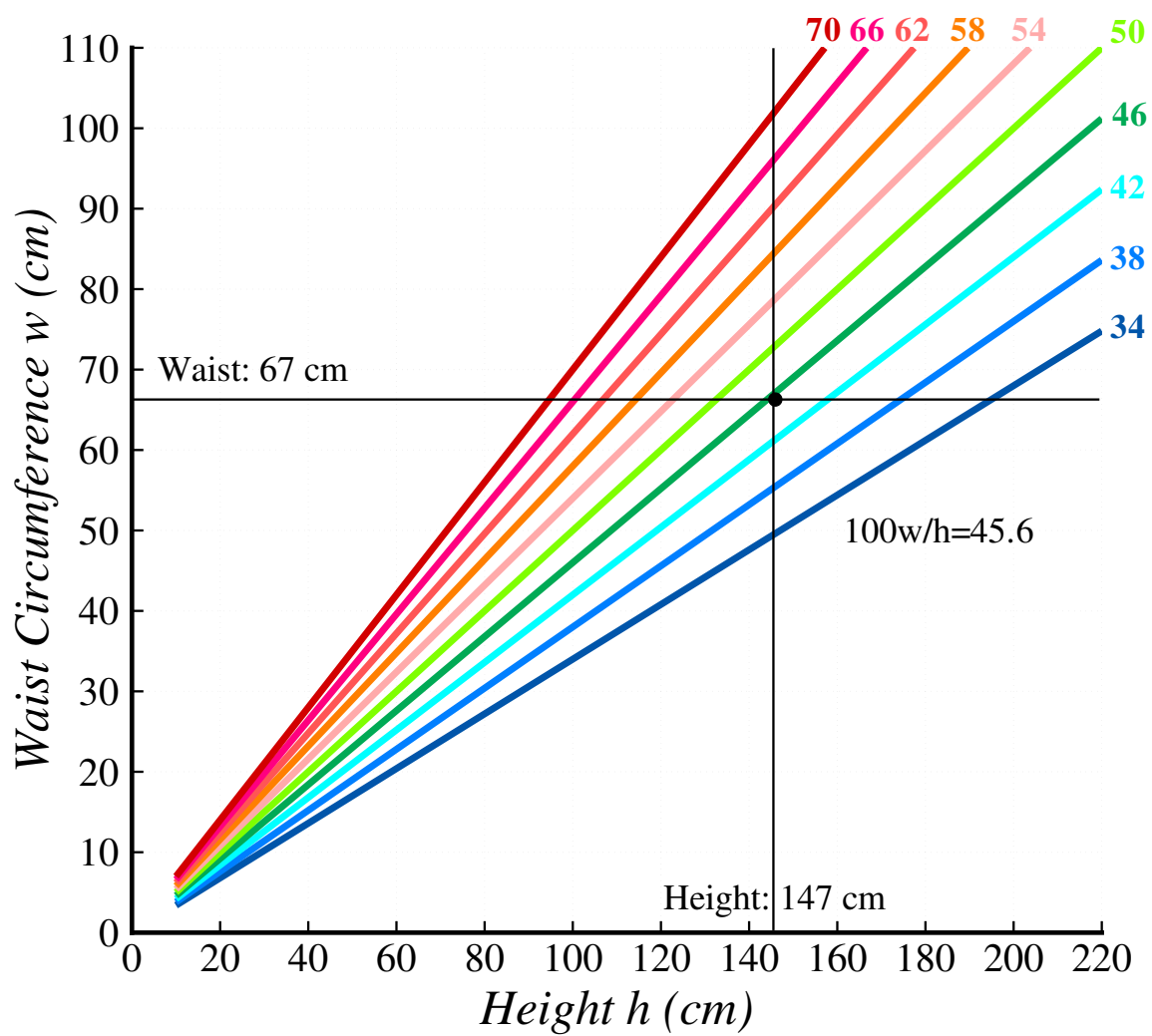
A sample graphical application is given in Figure 4.2 for a participant of the measurements: 10-year-old female with 40 kg mass, 147 cm height, 67 cm waist circumference. The waist-to-height ratio is  $100 \cdot w/h = 45.6$ , which is very slightly below the *mean normal* or *ideal* level of 46. The classic body mass index on the other hand gives  $BMI = 18.5 \text{ kg/m}^2$ , which is the lower limit of the normal range ( $18.5 \text{ kg/m}^2 - 24.9 \text{ kg/m}^2$ ), almost underweight.

## Criterion Based on Consistent Body Mass Index CBMI

The basic statistical characteristics of consistent body mass index CBMI for subgroups and entire measurements given in Table 3.3 shows that the overall average for  $CBMI = \sqrt{m/h^3}$  is 3.85 with standard deviation  $\pm 0.45$ , which may be rounded off to  $3.9 \pm 0.5$ . In line with the waist-to-height ratios the average CBMI value 3.9 is set to the upper limit of normal and the ranges are arranged according to the waist-to-height classification with



**Figure 4.1:** Classification of fitness levels according to 100-waist-to-height ratios. Interval between 42 to 50 normal levels with ideal average 46.



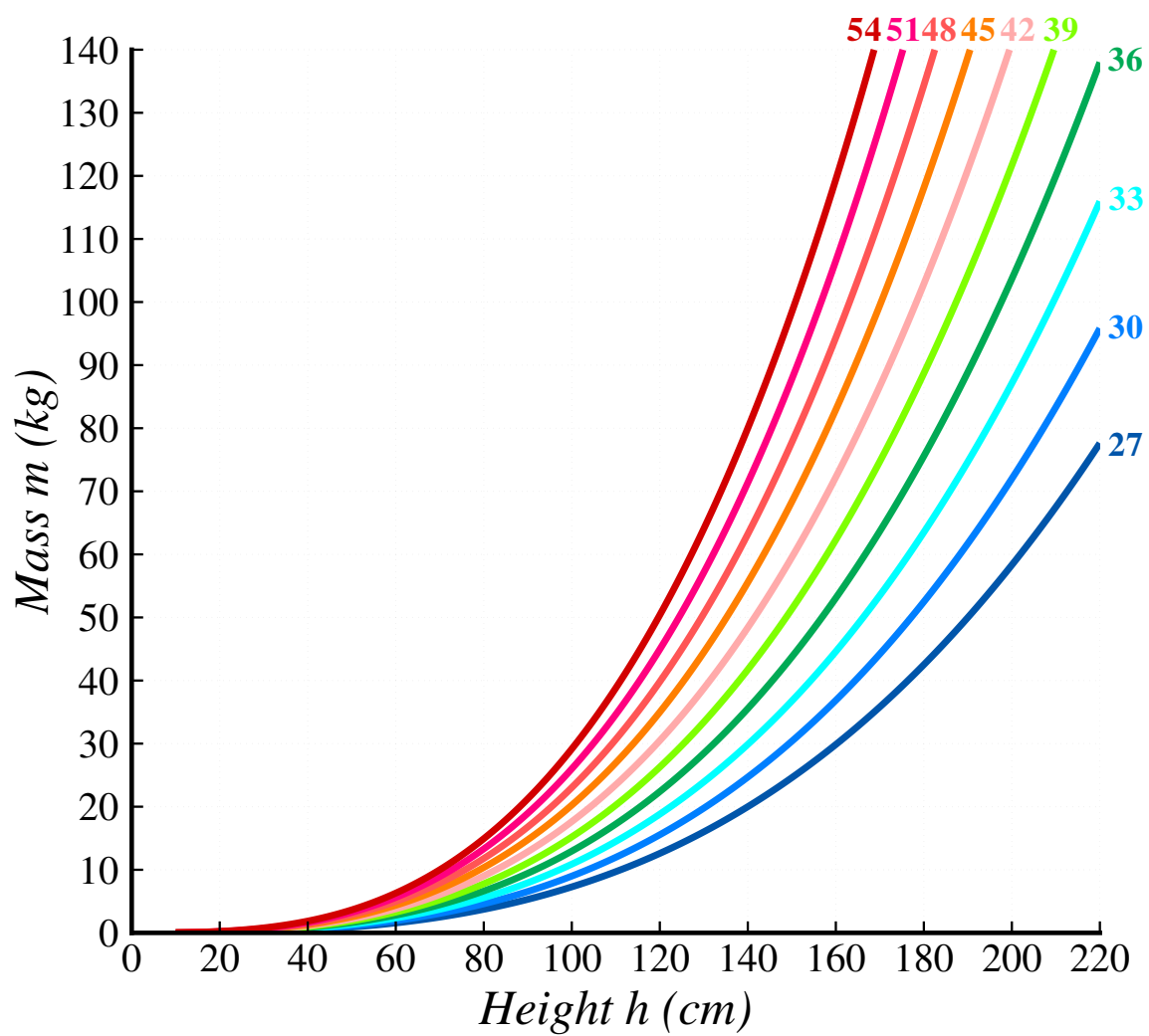
**Figure 4.2:** A sample graphical assessment of fitness level for a 10-year-old female: 147 cm height and 67 cm waist circumference. Assessed normal with  $100 \cdot w/h = 45.6$  nearly equal to ideal normal 46.

**Table 4.2:** Fitness levels according to  $10 \cdot \text{CBMI}$ , corresponding  $100 \cdot w/h$  as computed from  $100 \cdot \alpha \cdot \text{CBMI}$  with  $\alpha = 0.129$ , and  $100 \cdot w/h$  ranges as defined in Table 4.1.

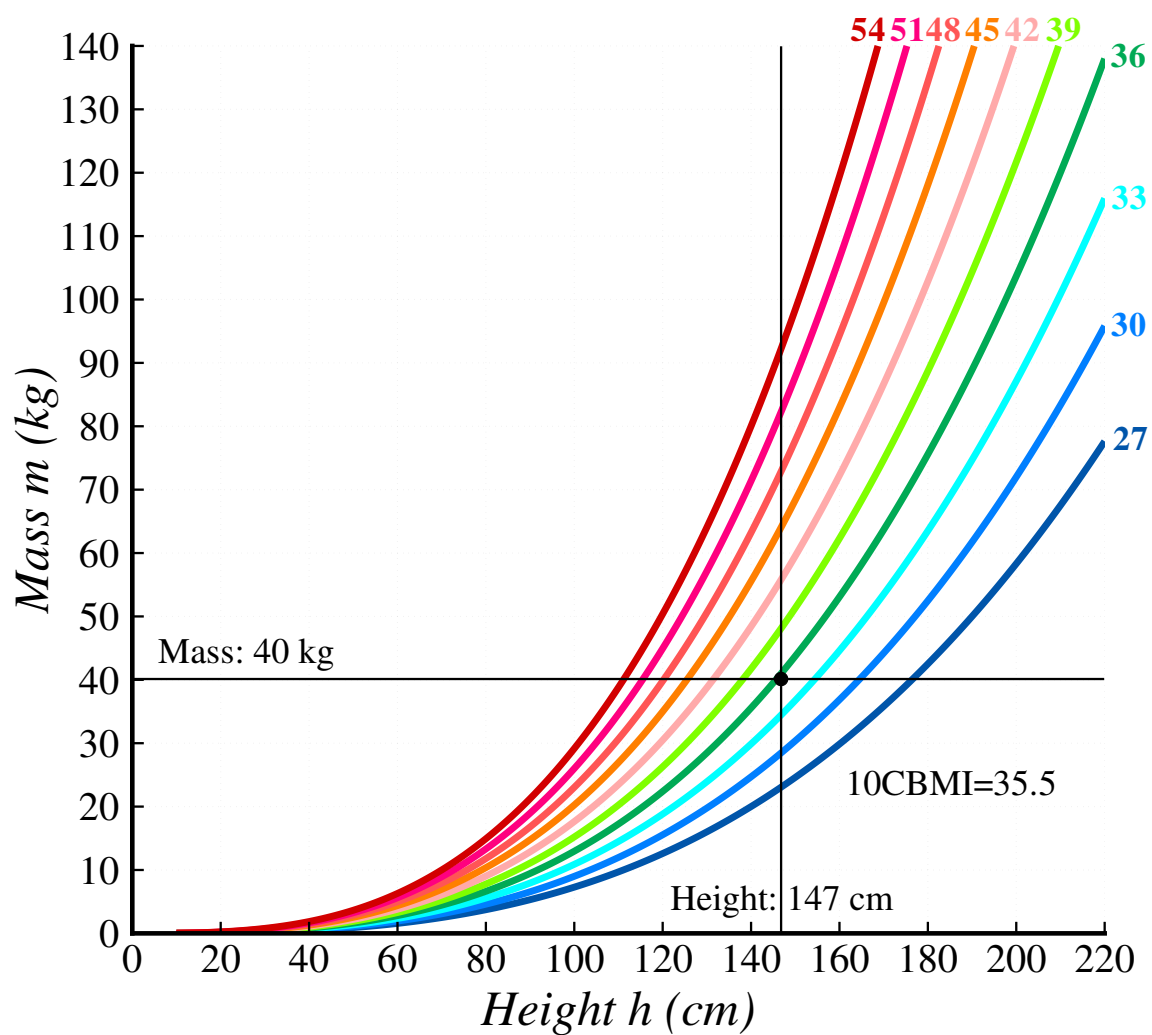
Classification	$10 \cdot \text{CBMI}$	$100 \cdot w/h = 100 \cdot \alpha \cdot \text{CBMI}$	$100 \cdot w/h$
Underweight	< 33	< 42.6	< 42
Normal	33 – 39	42.6 – 50.3	42 – 50
Overweight	39 – 45	50.3 – 58.1	50 – 58
Obese	45 – 51	58.1 – 65.8	58 – 66
Morbid obese	> 51	> 65.8	> 66

appropriate rounding of values. The ideal normal is then the mid-value  $\text{CBMI} = 3.6$  between 3.3–3.9. For the sake of using integers, the ranges are widened and  $10 \cdot \text{CBMI}$  is used in place of  $\text{CBMI}$ . Table 4.2 gives the classification levels according to  $\text{CBMI}$  values, the corresponding waist-to-height ratios as computed from  $100 \cdot w/h = 100 \cdot \alpha \cdot \text{CBMI}$  with  $\alpha = 0.129$ , and  $100 \cdot w/h$  ranges defined in Table 4.1. Clearly, the corresponding ranges of  $\text{CBMI}$  computed from  $100 \cdot \alpha \cdot \text{CBMI}$  agree very closely with the classification ranges of  $100 \cdot w/h$  set in Table 4.1. Visual representation of Table 4.2 is presented in Figure 4.3 where the fitness levels according to  $10 \cdot \text{CBMI}$  values are drawn.

$\text{CBMI}$  classification is graphically demonstrated in Figure 4.4 for the same 10-year-old female: 40 kg mass, 147 cm height, 67 cm waist circumference. Accordingly, the consistent body mass index is  $10 \cdot \text{CBMI} = 35.5$ , which, as observed for waist-to-height ratio, indicates very nearly perfect normal body fitness level of 36, based on the scale set above. Note that for this case the classic body mass index as calculated before gives  $\text{BMI} = 18.5 \text{ kg/m}^2$ , which is the lower limit of the normal range ( $18.5 \text{ kg/m}^2 - 24.9 \text{ kg/m}^2$ ) or the upper limit of the underweight category. Although it is not possible to claim absolute correctness for the present index, the consistency of it with waist-to-height ratio in making the same assessment for this particular example provides a confidence for its categorization. Furthermore,  $\text{BMI}$  is known to be unreliable when applied to relatively younger or shorter/taller individuals.



**Figure 4.3:** Classification of fitness levels according to  $10 \cdot \text{CBMI}$  values. Interval between 33 and 39 normal levels with ideal average 36.



**Figure 4.4:** A sample graphical assessment of fitness level for a 10-year-old female: 40 kg mass, 147 cm height. Assessed normal with  $10 \cdot \text{CBMI} = 35.5$  slightly less than ideal normal 36.

# 5

## Discussion

The present study has theoretically derived from first principles a new index termed Consistent Body Mass Index  $CBMI = \sqrt{m/h^3}$ , which is in accord with waist-to-height ratio  $w/h$ , a key parameter geometrically identifying body fitness level. The new index is non-dimensional, valid for all ages, body sizes, and sexes. General applicability of the new index and its satisfactory correlation with waist-to-height ratio render it a reliable and accurate measure of fitness level, thus providing a quantitative value for assessing health risks associated with underweight or overweight body states for all ages and sizes.

Besides serving as body fitness measures, body mass index  $BMI = m/h^2$  and triponderal mass index  $TMI = m/h^3$  are frequently used in pediatric growth assessment. However, evidence suggests that both measures misclassify adiposity, especially for children and relatively short or tall adults. BMI typically underestimates adiposity for shorter persons while overestimates for taller ones. The non-dimensional TMI overcomes the scalability issue but does not correlate with waist-to-height ratio, an important geometric measure. Systematic reviews and cohort comparisons consistently show that TMI performs similar to or slightly better than BMI in estimating body fat percentage; neither being completely reliable for all heights and ages [16]. Conversely, when compared with BMI, waist-to-height-ratio  $w/h$  is found to have better longitudinal agreement with fat-mass measured directly by Dual-Energy X-ray Absorptiometry DXA (DEXA) in a cohort of children and adolescents, a finding underscoring the limitations of BMI as a proxy for adiposity [17, 18]. TMI does not correlate with  $w/h$  for children and infants, especially for the higher range  $w/h > 0.6$  as demonstrated here. This finding is naturally interpreted as the failure of TMI for children and infants. Refined body composition phenotyping such as DXA, or Bioelectrical Impedance Analysis BIA, or related modalities should be used for accurate quantification of fat and fat-free compartments, especially for individuals with “normal-BMI, high-fat”, or relatively taller/shorter adults and children, or sarcopenic profiles, where BMI is the least informative [19]. However, these modalities are neither easily nor frequently applicable in practice.

To address and subsequently resolve the issues enumerated, a readily accessible body composition indicator, the consistent body mass index  $\text{CBMI} = \sqrt{m/h^3}$  is suggested here. Consistency of the new index is attributed to three main points: 1) it is non-dimensional, 2) it is directly proportional to  $w/h$ , and 3) it is applicable for all body sizes and age groups starting from newborns, regardless of sex. The existence of a direct proportionality between  $w/h$  and CBMI indicates that any one of these two quantities may be used as a measure of body fitness level.

Relatively low correlation coefficient  $r = 0.744$  of CBMI versus  $w/h$  for male adults, especially when compared with female children  $r = 0.907$ , is basically attributed to increase/decrease of body mass density  $\rho$  due to decrease/increase (lower/higher adipose) in  $w/h$  ratio. This hypothesis was tested by retaining the body density in the CBMI formulation as  $\text{CBMI} = \sqrt{m/\rho h^3}$  and empirically increasing mass density  $\rho$  for men with  $w/h < 0.5$ , keeping the same for  $w/h = 0.5$  (mean waist-to-height ratio), and decreasing for  $w/h > 0.5$ . This empirical rectification of  $\rho$  by approximately  $\pm 3\%$  on the average increased the correlation coefficient of the same data from  $r = 0.744$  to  $r = 0.800$  (7.5% improvement) for male adults. A similar trial performed for female adults increased the correlation coefficient from  $r = 0.840$  to  $r = 0.877$  (4.4% improvement). These results are encouraging for introducing a more accurate CBMI in a future work with the inclusion of  $\rho$  computed according to  $\rho = \rho(w/h)$  or equivalently  $\rho = \rho(\text{CBMI})$ . Finally, CBMI can also be used as an estimator for ideal body weight for a definite CBMI value and a given height  $h$  from  $m = (\text{CBMI})^2 h^3$ . Thus, studies concerned with general public health and well-being [8] may considerably benefit from the use of these fundamentally sound classification indexes, which are completely scalable hence universal. Conducting extensive measurement programs of weight, height, and waist circumference in different regions of the World would provide valuable data to establish accurate classification levels and assess relevant health risks.

# 6

## Contributions

**Serdar Beji** conceived the original idea for the study and did the theoretical formulation. Prepared the software, worked out the numerical and statistical details for correlation computations and visualizations. Drafted the original version of the report and reviewed the final report.

**Nezihe Kızılkaya Beji** arranged the ethical permission documents and the team for data collection. Curated the raw data and ensured its quality. Suggested references and commented on the statistical results and correlations. Reviewed the draft and the final version of the report.

**Ümmü Mutlu** identified the important main parameters and suggested references. Provided a preliminary data set for trial computations. Commented on the statistical results and correlations. Drafted and reviewed the final version of the report.

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# A

## Data

The data collected for the work is presented in the following order: Female adults, male adults, female children, and male children.

### Female Adults

	Age	Mass	Height	Waist	w/h	alpha*CBMI	sqrt(m/h^3)
1	10	26.00	126.00	63.00	0.500	0.465	3.605
2	10	27.00	135.00	55.00	0.407	0.427	3.313
3	10	29.00	134.00	63.00	0.470	0.448	3.472
4	10	40.00	147.00	67.00	0.456	0.458	3.549
5	10	40.00	137.00	58.00	0.423	0.509	3.944
6	11	32.00	148.00	60.00	0.405	0.405	3.142
7	11	34.00	143.00	62.00	0.434	0.440	3.410
8	11	41.00	155.00	63.00	0.406	0.428	3.318
9	13	47.30	160.00	67.00	0.419	0.438	3.398
10	13	48.00	165.00	72.00	0.436	0.422	3.269
11	13	57.00	155.00	81.00	0.523	0.505	3.912
12	14	56.00	172.00	67.00	0.390	0.428	3.317
13	14	70.00	168.00	87.00	0.518	0.496	3.842
14	15	40.00	155.00	69.00	0.445	0.423	3.277
15	15	56.00	176.00	69.00	0.392	0.413	3.205
16	15	65.00	167.00	81.00	0.485	0.482	3.736
17	17	55.00	178.00	78.00	0.438	0.403	3.123
18	17	68.00	180.00	84.00	0.467	0.440	3.415
19	19	56.00	171.00	89.00	0.520	0.432	3.347
20	20	63.00	159.00	75.00	0.472	0.511	3.959
21	20	57.00	178.00	68.00	0.382	0.410	3.179
22	22	52.00	163.00	72.00	0.442	0.447	3.465
23	22	61.20	172.00	65.00	0.378	0.447	3.468
24	22	65.00	165.00	65.00	0.394	0.491	3.804
25	23	53.00	168.00	72.50	0.432	0.431	3.343
26	23	60.00	157.00	72.00	0.459	0.508	3.938
27	24	48.00	155.00	63.00	0.406	0.463	3.590
28	24	66.00	164.00	74.00	0.451	0.499	3.868
29	24	57.00	170.00	97.00	0.571	0.439	3.406
30	25	62.00	167.00	112.00	0.671	0.471	3.649
31	25	96.00	156.00	74.00	0.474	0.649	5.029

33	25	62.00	167.00	74.00	0.443	0.471	3.649
34	25	62.00	161.00	93.00	0.578	0.497	3.854
35	25	66.00	164.00	92.00	0.561	0.499	3.868
36	26	63.00	178.00	72.00	0.404	0.431	3.342
37	26	54.00	157.00	70.00	0.446	0.482	3.735
38	26	58.00	165.00	74.00	0.448	0.464	3.593
39	26	68.00	162.00	106.00	0.654	0.516	3.999
40	27	58.00	166.00	104.00	0.627	0.459	3.561
41	27	62.00	163.00	70.00	0.429	0.488	3.784
42	27	50.00	160.00	74.00	0.463	0.451	3.494
43	28	60.00	165.00	90.00	0.545	0.471	3.655
44	28	64.00	165.00	71.00	0.430	0.487	3.775
45	28	89.00	176.00	69.00	0.392	0.521	4.040
46	28	94.00	168.00	104.00	0.619	0.574	4.452
47	28	60.00	165.00	76.00	0.461	0.471	3.655
48	28	64.00	165.00	64.00	0.388	0.487	3.775
49	28	55.00	158.00	78.00	0.494	0.482	3.734
50	28	54.00	157.00	98.00	0.624	0.482	3.735
51	29	65.00	162.00	78.00	0.481	0.504	3.910
52	29	67.00	163.00	66.00	0.405	0.507	3.933
53	29	52.00	165.00	81.00	0.491	0.439	3.402
54	29	62.00	167.00	72.00	0.431	0.471	3.649
55	30	101.00	178.00	86.00	0.483	0.546	4.232
56	30	57.00	159.00	79.00	0.497	0.486	3.766
57	30	70.00	157.00	85.00	0.541	0.549	4.253
58	31	84.00	177.00	82.00	0.463	0.502	3.892
59	32	67.00	163.00	86.00	0.528	0.507	3.933
60	32	67.60	169.00	112.00	0.663	0.483	3.742
61	33	50.70	155.00	105.00	0.677	0.476	3.690
62	33	58.00	171.00	68.00	0.398	0.439	3.406
63	33	54.00	158.00	68.00	0.430	0.477	3.700
64	33	85.00	156.00	59.00	0.378	0.610	4.732
65	33	64.00	160.00	78.00	0.488	0.510	3.953
66	34	72.00	165.00	73.00	0.442	0.516	4.004
67	34	55.00	163.00	90.00	0.552	0.460	3.564
68	34	62.00	164.00	85.00	0.518	0.484	3.749
69	35	53.00	158.00	72.00	0.456	0.473	3.666
70	35	65.00	152.00	91.00	0.599	0.555	4.302
71	35	105.00	159.00	78.00	0.491	0.659	5.111
72	35	82.00	170.00	78.00	0.459	0.527	4.085
73	36	65.00	157.00	99.00	0.631	0.529	4.098
74	36	58.00	163.00	86.00	0.528	0.472	3.660
75	36	63.00	159.00	98.00	0.616	0.511	3.959
76	36	53.00	155.00	80.00	0.516	0.487	3.773
77	36	55.00	166.00	87.00	0.524	0.447	3.468
78	37	64.00	171.00	66.00	0.386	0.462	3.578
79	38	57.00	156.00	85.00	0.545	0.500	3.875
80	38	70.00	174.00	66.00	0.379	0.470	3.645
81	38	50.00	167.00	85.00	0.509	0.423	3.277
82	39	81.00	174.00	83.00	0.477	0.506	3.921
83	40	65.00	164.00	82.00	0.500	0.495	3.839
84	40	88.00	163.00	69.00	0.423	0.581	4.508
85	40	95.00	167.00	103.00	0.617	0.583	4.516
86	40	92.60	181.00	91.00	0.503	0.510	3.952
87	40	94.00	190.00	94.00	0.495	0.478	3.702
88	41	67.00	163.00	61.00	0.374	0.507	3.933

89	41	87.00	167.00	87.00	0.521	0.558	4.322	
90	41	66.00	160.00	96.00	0.600	0.518	4.014	
91	42	54.00	160.00	108.00	0.675	0.468	3.631	
92	42	82.00	154.00	96.00	0.623	0.611	4.738	
93	43	58.00	163.00	107.00	0.656	0.472	3.660	
94	44	54.00	155.00	88.00	0.568	0.491	3.808	
95	45	65.00	175.00	82.00	0.469	0.449	3.483	
96	45	65.00	169.00	96.00	0.568	0.473	3.670	
97	48	68.00	152.00	82.00	0.539	0.568	4.400	
98	59	77.00	156.00	81.00	0.519	0.581	4.504	
99	62	62.00	157.00	70.00	0.446	0.516	4.003	
100	62	59.00	156.00	98.00	0.628	0.509	3.942	
101	70	65.00	158.00	80.00	0.506	0.524	4.059	
102								
103	10	26.00	126.00	55.00	0.374	0.403	3.123	MIN
104	70	105.00	190.00	112.00	0.677	0.659	5.111	MAX
105	30	62.48	162.51	80.02	0.493	0.488	3.786	AVG
106	12	15.30	10.00	13.50	0.081	0.050	0.388	STD

### Male Adults

	Age	Mass	Height	Waist	w/h	alpha*CBMI	sqrt(m/h^3)	
1	10	23.00	128.00	62.00	0.484	0.430	3.312	
2	10	25.00	130.00	63.00	0.485	0.438	3.373	
3	10	25.00	126.00	62.00	0.492	0.459	3.535	
4	10	26.00	129.00	63.00	0.488	0.452	3.480	
5	10	30.00	132.00	60.00	0.455	0.469	3.612	
6	10	30.00	143.00	60.00	0.420	0.416	3.203	
7	10	33.00	140.00	60.00	0.429	0.450	3.468	
8	10	35.00	147.00	64.00	0.435	0.431	3.319	
9	10	35.00	140.00	60.00	0.429	0.464	3.571	
10	11	32.00	140.00	57.00	0.407	0.443	3.415	
11	11	33.00	138.00	61.00	0.442	0.460	3.544	
12	11	43.00	151.00	71.00	0.470	0.459	3.534	
13	11	33.00	142.00	57.00	0.401	0.441	3.395	
14	13	48.00	165.00	72.00	0.436	0.425	3.269	
15	13	48.40	158.00	66.00	0.418	0.455	3.503	
16	13	57.00	155.00	80.00	0.516	0.508	3.912	
17	14	40.00	155.00	69.00	0.445	0.426	3.277	
18	14	52.50	172.00	68.00	0.395	0.417	3.212	
19	14	70.00	168.00	87.00	0.518	0.499	3.842	
20	14	74.00	170.00	94.00	0.553	0.504	3.881	
21	14	78.00	163.00	90.00	0.552	0.551	4.244	
22	16	46.00	161.00	63.00	0.391	0.431	3.320	
23	16	66.00	174.00	80.00	0.460	0.460	3.540	
24	16	75.00	170.00	94.00	0.553	0.507	3.907	
25	17	55.00	178.00	78.00	0.438	0.406	3.123	
26	17	68.00	180.00	84.00	0.467	0.443	3.415	
27	19	68.00	177.00	68.00	0.384	0.455	3.502	
28	19	90.00	192.00	92.00	0.479	0.463	3.566	
29	20	64.00	168.00	70.00	0.417	0.477	3.674	
30	20	75.00	171.00	90.00	0.526	0.503	3.873	
31	21	79.00	177.00	90.00	0.508	0.490	3.774	
32	22	51.80	164.00	63.00	0.384	0.445	3.427	
33	22	55.00	165.00	75.00	0.455	0.454	3.499	

35	22	56.00	158.00	64.00	0.405	0.489	3.768
36	22	77.00	175.00	85.00	0.486	0.492	3.790
37	23	62.00	155.00	77.00	0.497	0.530	4.080
38	23	90.00	187.00	81.00	0.433	0.482	3.710
39	23	90.00	187.00	76.00	0.406	0.482	3.710
40	23	97.00	180.00	102.00	0.567	0.530	4.078
41	24	74.00	179.00	102.00	0.570	0.466	3.592
42	25	65.00	168.00	73.00	0.435	0.481	3.702
43	25	72.00	170.00	90.00	0.529	0.497	3.828
44	25	73.00	168.00	89.00	0.530	0.510	3.924
45	25	61.00	168.00	77.00	0.458	0.466	3.587
46	25	88.00	181.00	91.00	0.503	0.500	3.852
47	26	62.00	173.00	68.00	0.393	0.449	3.460
48	28	54.00	156.00	68.00	0.436	0.490	3.771
49	28	78.00	180.00	88.00	0.489	0.475	3.657
50	29	53.00	150.00	70.00	0.467	0.515	3.963
51	30	80.00	180.00	88.00	0.489	0.481	3.704
52	30	78.00	175.00	90.00	0.514	0.495	3.815
53	30	110.00	190.00	110.00	0.579	0.520	4.005
54	31	71.50	177.00	73.00	0.412	0.466	3.591
55	33	87.00	179.00	95.00	0.531	0.506	3.895
56	34	44.00	150.00	57.00	0.380	0.469	3.611
57	34	77.40	173.00	85.00	0.491	0.502	3.866
58	35	67.00	162.00	99.00	0.611	0.516	3.970
59	35	82.00	178.00	92.00	0.517	0.495	3.813
60	35	85.00	173.00	96.00	0.555	0.526	4.052
61	35	86.00	182.00	83.00	0.456	0.491	3.777
62	35	70.00	175.00	88.00	0.503	0.469	3.614
63	35	78.00	171.00	98.00	0.573	0.513	3.950
64	35	82.00	190.00	87.00	0.458	0.449	3.458
65	36	58.00	172.00	68.00	0.395	0.438	3.376
66	36	86.00	177.00	89.00	0.503	0.511	3.938
67	37	79.00	175.00	95.00	0.543	0.499	3.839
68	37	79.00	175.00	95.00	0.543	0.499	3.839
69	39	96.00	180.00	92.00	0.511	0.527	4.057
70	42	90.00	182.00	95.00	0.522	0.502	3.864
71	42	75.00	172.00	89.00	0.517	0.499	3.839
72	42	90.00	180.00	99.00	0.550	0.510	3.928
73	43	95.00	186.00	102.00	0.548	0.499	3.842
74	45	67.00	161.00	78.00	0.484	0.520	4.007
75	45	90.00	180.00	98.00	0.544	0.510	3.928
76	45	70.00	172.00	88.00	0.512	0.482	3.709
77	47	85.00	170.00	104.00	0.612	0.540	4.159
78	52	73.00	166.00	88.00	0.530	0.519	3.995
79	52	77.00	180.00	88.00	0.489	0.472	3.634
80	52	87.00	173.00	102.00	0.590	0.532	4.099
81	52	88.00	181.00	104.00	0.575	0.500	3.852
82	53	150.00	205.00	124.00	0.605	0.542	4.173
83	53	80.00	173.00	94.00	0.543	0.510	3.931
84	55	83.00	178.00	101.00	0.567	0.498	3.836
85	58	57.00	162.00	71.00	0.438	0.476	3.662
86	58	58.00	160.00	89.00	0.556	0.489	3.763
87	58	74.20	163.00	90.00	0.552	0.538	4.139
88	60	78.00	162.00	88.00	0.543	0.556	4.283
89	60	90.00	174.00	105.00	0.603	0.537	4.133
90	61	64.00	176.00	91.00	0.517	0.445	3.426

91	62	58.70	155.00	75.00	0.484	0.516	3.970	
92	62	84.00	164.00	103.00	0.628	0.567	4.364	
93	63	79.00	174.00	93.00	0.534	0.503	3.872	
94	63	93.00	175.00	109.00	0.623	0.541	4.166	
95	64	83.00	178.00	93.00	0.522	0.498	3.836	
96	65	70.00	174.00	88.00	0.506	0.473	3.645	
97	66	81.00	165.00	103.00	0.624	0.551	4.246	
98	67	90.00	190.00	90.00	0.474	0.470	3.622	
99	68	71.50	173.00	96.00	0.555	0.483	3.716	
100	70	83.00	165.50	108.00	0.653	0.556	4.279	
101	75	72.00	170.00	95.00	0.559	0.497	3.828	
102								
103	10	23.00	126.00	57.00	0.38	0.406	3.123	MIN
104	75	150.00	205.00	124.00	0.65	0.567	4.364	MAX
105	33	68.98	167.78	83.95	0.50	0.487	3.749	AVG
106	18	21.15	15.24	14.89	0.06	0.035	0.273	STD

### Female Children

1	Age	Mass	Height	Waist	w/h	alpha*CBMI	sqrt(m/h^3)
2	0.0	1.78	41.00	23.40	0.571	0.661	5.082
3	0.0	3.10	51.00	32.00	0.627	0.628	4.834
4	0.0	3.55	52.00	30.00	0.577	0.653	5.021
5	0.0	4.10	52.00	32.00	0.615	0.702	5.400
6	0.0	4.40	55.00	33.00	0.600	0.669	5.143
7	0.0	2.66	48.00	30.00	0.625	0.638	4.904
8	0.0	3.20	48.00	34.50	0.719	0.699	5.379
9	0.0	3.35	52.00	34.00	0.654	0.635	4.881
10	0.0	3.40	51.00	33.50	0.657	0.658	5.063
11	0.5	6.00	65.00	42.00	0.646	0.608	4.674
12	0.5	7.30	66.50	42.00	0.632	0.648	4.982
13	0.8	9.00	73.00	45.00	0.616	0.625	4.810
14	0.9	9.40	76.00	46.00	0.605	0.602	4.627
15	1.0	9.60	73.00	49.00	0.671	0.646	4.968
16	1.0	11.00	81.00	49.00	0.605	0.591	4.550
17	1.0	11.00	81.00	49.00	0.605	0.591	4.550
18	1.2	10.00	77.00	47.00	0.610	0.608	4.680
19	1.5	9.50	75.00	47.60	0.635	0.617	4.745
20	1.5	10.20	78.00	47.00	0.603	0.603	4.636
21	2	10.50	85.00	48.00	0.565	0.538	4.135
22	2	10.50	78.00	42.00	0.538	0.611	4.704
23	2	11.00	95.00	42.00	0.442	0.466	3.582
24	2	11.50	97.00	43.50	0.448	0.461	3.550
25	2	12.00	85.00	50.00	0.588	0.575	4.420
26	2	12.50	88.00	46.00	0.523	0.557	4.283
27	2	12.50	100.00	47.00	0.470	0.460	3.536
28	2	11.40	83.00	48.00	0.578	0.580	4.465
29	2	12.00	88.00	50.00	0.568	0.546	4.196
30	3	12.20	95.00	50.00	0.526	0.490	3.772
31	3	13.00	93.00	52.00	0.559	0.523	4.020
32	3	13.00	94.00	49.00	0.521	0.514	3.956
33	3	13.00	95.00	48.00	0.505	0.506	3.894
34	3	13.40	95.00	49.00	0.516	0.514	3.953
35	3	14.00	95.00	49.00	0.516	0.525	4.041
36	3	15.00	96.00	50.00	0.521	0.535	4.118

37	3	16.00	98.00	52.00	0.531	0.536	4.123
38	3	18.00	87.80	50.00	0.569	0.670	5.157
39	3	13.40	95.00	49.00	0.516	0.514	3.953
40	4	15.00	96.00	51.00	0.531	0.535	4.118
41	4	15.00	104.00	50.00	0.481	0.475	3.652
42	4	15.00	98.00	50.00	0.510	0.519	3.992
43	4	17.00	99.00	54.00	0.545	0.544	4.186
44	4	18.00	108.00	52.00	0.481	0.491	3.780
45	4	19.00	100.00	54.00	0.540	0.567	4.359
46	5	17.00	109.00	51.00	0.468	0.471	3.623
47	5	18.00	107.00	46.00	0.430	0.498	3.833
48	5	18.50	112.00	54.00	0.482	0.472	3.629
49	5	19.00	111.00	52.00	0.468	0.485	3.727
50	5	19.00	110.00	59.00	0.536	0.491	3.778
51	5	19.00	108.00	52.00	0.481	0.505	3.884
52	5	20.00	110.00	51.00	0.464	0.504	3.876
53	6	15.00	100.00	48.00	0.480	0.503	3.873
54	6	17.00	110.00	52.00	0.473	0.465	3.574
55	6	17.40	107.00	56.00	0.523	0.490	3.769
56	6	18.00	110.00	51.00	0.464	0.478	3.677
57	6	18.00	111.00	53.00	0.477	0.472	3.628
58	6	19.00	112.00	53.00	0.473	0.478	3.677
59	6	19.00	115.00	53.00	0.461	0.459	3.535
60	6	20.00	114.00	52.00	0.456	0.478	3.674
61	6	20.00	117.00	51.00	0.436	0.459	3.534
62	6	21.00	115.00	54.00	0.470	0.483	3.716
63	6	21.00	119.00	51.00	0.429	0.459	3.530
64	6	23.00	119.00	57.00	0.479	0.480	3.694
65	6	30.00	122.00	57.00	0.467	0.528	4.065
66	7	19.00	115.00	54.00	0.470	0.459	3.535
67	7	20.00	116.00	55.00	0.474	0.465	3.580
68	7	20.00	114.00	54.00	0.474	0.478	3.674
69	7	21.00	119.00	54.00	0.454	0.459	3.530
70	7	21.00	120.00	53.00	0.442	0.453	3.486
71	7	21.00	118.00	56.00	0.475	0.465	3.575
72	7	21.00	119.00	56.00	0.471	0.459	3.530
73	7	21.00	117.00	55.00	0.470	0.471	3.621
74	7	22.00	118.00	58.00	0.492	0.476	3.659
75	7	23.25	129.00	58.00	0.450	0.428	3.291
76	7	24.00	132.00	50.00	0.379	0.420	3.230
77	7	25.00	122.00	58.00	0.475	0.482	3.710
78	7	25.00	126.00	57.50	0.456	0.460	3.535
79	7	26.00	122.00	59.00	0.484	0.492	3.784
80	7	38.00	140.00	66.00	0.471	0.484	3.721
81	8	23.00	120.00	58.00	0.483	0.474	3.648
82	8	24.00	122.00	61.00	0.500	0.473	3.636
83	8	26.00	131.00	57.00	0.435	0.442	3.401
84	8	27.00	130.00	60.00	0.462	0.456	3.506
85	8	27.00	128.00	58.00	0.453	0.466	3.588
86	8	28.00	128.00	58.00	0.453	0.475	3.654
87	8	29.00	138.00	60.00	0.435	0.432	3.322
88	8	29.00	125.00	62.00	0.496	0.501	3.853
89	8	35.00	132.00	69.00	0.523	0.507	3.901
90	9	24.00	128.00	58.00	0.453	0.440	3.383
91	9	24.00	128.00	62.00	0.484	0.440	3.383
92	9	24.00	127.00	60.00	0.472	0.445	3.423

93	9	25.00	132.00	52.00	0.394	0.429	3.297	
94	9	25.00	127.00	61.00	0.480	0.454	3.494	
95	9	25.00	125.00	60.00	0.480	0.465	3.578	
96	9	25.00	127.00	61.00	0.480	0.454	3.494	
97	9	26.00	128.00	62.00	0.484	0.458	3.521	
98	9	30.00	130.00	63.00	0.485	0.480	3.695	
99	9	30.00	130.00	56.00	0.431	0.480	3.695	
100	9	31.00	134.00	60.00	0.448	0.467	3.589	
101	9	32.00	137.00	56.00	0.409	0.459	3.528	
102								
103	0	1.78	41.00	23.40	0.379	0.420	3.230	MIN
104	9	38.00	140.00	69.00	0.719	0.702	5.400	MAX
105	5	17.72	103.15	51.22	0.511	0.516	3.972	AVG
106	3	7.89	24.51	8.29	0.068	0.071	0.545	STD

### Male Children

1	Age	Mass	Height	Waist	w/h	alpha*CBMI	sqrt(m/h^3)
2	0	2.87	46.00	31.00	0.674	0.706	5.430
3	0	3.30	52.00	33.00	0.635	0.630	4.845
4	0	3.50	50.00	35.50	0.710	0.688	5.292
5	0.5	7.30	66.50	42.00	0.632	0.648	4.982
6	0.5	8.20	69.00	43.00	0.623	0.649	4.996
7	0.9	9.40	76.00	46.00	0.605	0.602	4.627
8	1	11.00	72.00	50.00	0.694	0.706	5.429
9	1	12.70	74.00	47.00	0.635	0.728	5.598
10	1.5	11.50	83.00	44.00	0.530	0.583	4.485
11	2	11.50	85.00	47.00	0.553	0.563	4.327
12	2	12.10	84.00	46.00	0.548	0.587	4.518
13	2	12.50	100.00	52.00	0.520	0.460	3.536
14	2	14.00	96.00	48.00	0.500	0.517	3.978
15	2	14.00	96.00	49.00	0.510	0.517	3.978
16	2	15.50	105.00	48.00	0.457	0.476	3.659
17	2	15.60	88.00	54.00	0.614	0.622	4.785
18	2.5	13.40	98.00	53.00	0.541	0.491	3.773
19	2.5	14.50	95.00	46.00	0.484	0.535	4.112
20	2.5	16.70	88.00	52.00	0.591	0.644	4.950
21	2.5	16.70	88.00	50.00	0.568	0.644	4.950
22	3	12.40	94.00	51.00	0.543	0.502	3.864
23	3	12.50	94.00	51.00	0.543	0.504	3.879
24	3	13.00	95.00	53.00	0.558	0.506	3.894
25	3	13.50	92.00	50.00	0.543	0.541	4.164
26	3	14.20	95.00	51.00	0.537	0.529	4.070
27	3	14.20	95.00	50.00	0.526	0.529	4.070
28	3	16.00	97.00	58.00	0.598	0.544	4.187
29	3.5	15.00	83.00	51.00	0.614	0.666	5.122
30	4	15.00	98.00	50.00	0.510	0.519	3.992
31	4	16.00	103.00	58.00	0.563	0.497	3.827
32	4	16.00	102.00	52.00	0.510	0.505	3.883
33	4	16.00	102.00	48.00	0.471	0.505	3.883
34	5	16.00	105.00	50.00	0.476	0.483	3.718
35	5	17.00	108.00	50.00	0.463	0.478	3.674
36	5	17.00	109.00	51.00	0.468	0.471	3.623
37	5	18.00	108.00	50.00	0.463	0.491	3.780
38	5	19.00	108.00	51.50	0.477	0.505	3.884

39	5	19.00	108.00	55.00	0.509	0.505	3.884
40	5	19.00	111.00	52.00	0.468	0.485	3.727
41	5	22.00	111.00	58.00	0.523	0.521	4.011
42	5	19.00	112.00	50.00	0.446	0.478	3.677
43	5.5	15.00	109.00	52.00	0.477	0.442	3.403
44	5.5	17.50	109.00	54.00	0.495	0.478	3.676
45	6	21.00	117.00	55.00	0.470	0.471	3.621
46	6	18.00	112.00	52.00	0.464	0.465	3.579
47	6	18.00	110.00	52.00	0.473	0.478	3.677
48	6	18.00	111.00	52.00	0.468	0.472	3.628
49	6	19.00	113.00	53.00	0.469	0.472	3.629
50	6	19.00	110.00	52.00	0.473	0.491	3.778
51	6	20.00	114.00	53.00	0.465	0.478	3.674
52	6	20.00	113.00	54.00	0.478	0.484	3.723
53	6	20.00	115.00	52.00	0.452	0.471	3.626
54	6	21.00	115.00	54.00	0.470	0.483	3.716
55	6	20.00	118.00	52.00	0.441	0.454	3.489
56	6	20.00	120.00	54.00	0.450	0.442	3.402
57	6.5	22.00	119.00	56.00	0.471	0.470	3.613
58	7	20.00	117.00	55.00	0.470	0.459	3.534
59	7	21.00	120.00	54.00	0.450	0.453	3.486
60	7	21.00	118.00	57.00	0.483	0.465	3.575
61	7	22.00	119.00	55.00	0.462	0.470	3.613
62	7	22.00	119.00	59.00	0.496	0.470	3.613
63	7	22.00	118.00	57.00	0.483	0.476	3.659
64	7	25.00	119.00	56.00	0.471	0.501	3.852
65	7	26.00	121.00	56.00	0.463	0.498	3.831
66	7	28.00	119.00	58.00	0.487	0.530	4.076
67	7	25.00	126.00	56.00	0.444	0.460	3.535
68	7.5	24.00	122.00	54.00	0.443	0.473	3.636
69	7.5	27.00	125.00	57.00	0.456	0.483	3.718
70	8	26.00	130.00	56.00	0.431	0.447	3.440
71	8	25.00	128.00	53.00	0.414	0.449	3.453
72	8	23.00	130.00	56.00	0.431	0.421	3.236
73	8	23.00	121.00	59.00	0.488	0.468	3.603
74	8	23.00	120.00	57.00	0.475	0.474	3.648
75	8	23.00	123.00	59.00	0.480	0.457	3.516
76	8	24.00	123.00	59.00	0.480	0.467	3.591
77	8	24.00	122.00	59.00	0.484	0.473	3.636
78	8	35.00	132.00	69.00	0.523	0.507	3.901
79	8	25.00	122.00	60.00	0.492	0.482	3.710
80	8	27.00	133.00	58.00	0.436	0.440	3.388
81	8	26.00	128.00	56.00	0.438	0.458	3.521
82	8	28.00	127.00	56.00	0.441	0.481	3.697
83	8	32.00	124.00	58.00	0.468	0.533	4.097
84	8	29.00	125.00	57.00	0.456	0.501	3.853
85	8.5	37.00	129.00	60.00	0.465	0.540	4.152
86	8.5	30.00	132.00	58.00	0.439	0.470	3.612
87	9	23.00	133.00	57.20	0.430	0.406	3.127
88	9	23.00	133.00	61.00	0.459	0.406	3.127
89	9	23.00	125.00	60.00	0.480	0.446	3.432
90	9	24.00	124.00	60.00	0.484	0.461	3.548
91	9	24.00	124.00	60.00	0.484	0.461	3.548
92	9	25.00	136.00	57.00	0.419	0.410	3.153
93	9	25.00	127.00	61.00	0.480	0.454	3.494
94	9	28.00	139.00	56.00	0.403	0.420	3.229

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95	9	30.00	132.00	60.00	0.455	0.470	3.612	
96	9	30.00	134.00	59.00	0.440	0.459	3.531	
97	9	32.00	135.00	60.00	0.444	0.469	3.606	
98	9	43.00	146.00	82.00	0.562	0.483	3.717	
99	9	44.00	141.00	75.00	0.532	0.515	3.962	
100	9	50.00	151.00	85.00	0.563	0.495	3.811	
101	9	40.00	134.00	63.00	0.470	0.530	4.077	
102								
103	0	2.87	46.00	31.00	0.403	0.406	3.127	MIN
104	9	50.00	151.00	85.00	0.710	0.728	5.598	MAX
105	6	20.62	110.53	54.24	0.500	0.505	3.885	AVG
106	3	8.17	20.58	7.49	0.063	0.068	0.521	STD

# B

## FORTRAN Code for Least-Squares and Statistics

```
1 real m,h,w
2 C
3 C   Abbreviations: dfa=data of female adults, etc.
4   open(10,file='dfa')
5 C   open(10,file='dma')
6 C   open(10,file='dfc')
7 C   open(10,file='dmc')
8 C   open(10,file='dall')
9 C
10  open(40,file='out.plt')
11 C
12  n=100
13 C  n=400
14 C
15  smm=0; smw=0; sww=0
16 C
17  do j=1,n
18 C
19  read(10,*)m,h,w
20 C
21  h=h/100; w=w/100
22 C
23 C   CBMI
24  rm=sqrt(m/h/h/h); rw=w/h
25 C
26 C   TMI
27  rm=m/h/h/h; rw=w/h
28 C
29  smm=smm+rm*rm; smw=smw+rw*rm
30  sww=sww+rw*rw
31 C
32  enddo
33 C
34 C   Form factor (Proportionality coefficient)
35  alpha=smw/smm
36 C
```

```

37 write(*,*)"Form_factor",alpha
38 C
39 rewind(10)
40 C
41 smmean=0; swmean=0
42 C
43 do j=1,n
44 C
45 read(10,*)m,h,w
46 C
47 h=h/100; w=w/100
48 C
49 alpha*CBMI
50 rm=alpha*sqrt(m/h/h/h); rw=w/h
51 C
52 beta*TMI (beta for TMI is named alpha in computations for ease)
53 rm=alpha*m/h/h/h; rw=w/h
54 C
55 smmean=smmean+rm
56 swmean=swmean+rw
57 C
58 enddo
59 C
60 smmean=smmean/float(n)
61 swmean=swmean/float(n)
62 C
63 rewind(10)
64 C
65 sm=0; sw=0; smm=0; smw=0; sww=0
66 C
67 do k=1,n
68 C
69 read(10,*)m,h,w
70 C
71 h=h/100; w=w/100
72 C
73 alpha*CBMI
74 rm=alpha*sqrt(m/h/h/h); rw=w/h
75 C
76 beta*TMI
77 rm=alpha*m/h/h/h; rw=w/h
78 C
79 write(40,50)rw,rm
80 C
81 Residual error
82 sr=sr+(rm-rw)*(rm-rw)
83 C
84 Residual error of data points
85 st=st+(rw-swmean)*(rw-swmean)
86 C
87 Summations
88 sm=sm+rm; sw=sw+rw; smm=smm+rm*rm; smw=smw+rm*rw; sww=sww+rw*rw
89 C
90 enddo
91 C
92 write(*,*)"sr_and_st",sr,st,sr/st

```

```
93 C
94 C   Correlation coefficient
95     r=sqrt(1-sr/st)
96     write(*,*)"Correlation_coefficient",r
97     write(*,*)"Coefficient_of_variation",100*r*r
98 C
99 50   format(5(2x,f12.5))
100 C
101     stop
102     end
```